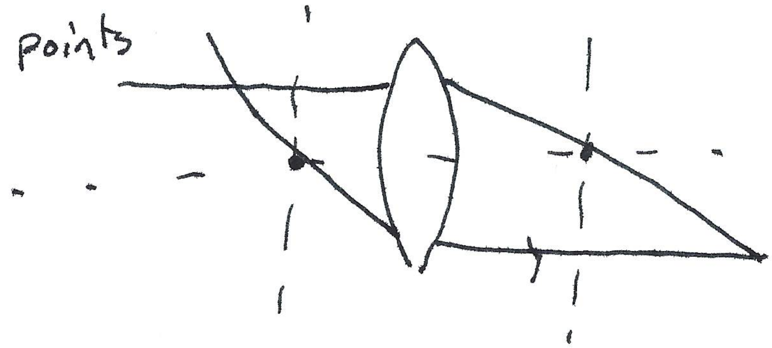
Phys 110B 09Dec20

Ray-tracing (Paraxial)

Cardinal Points

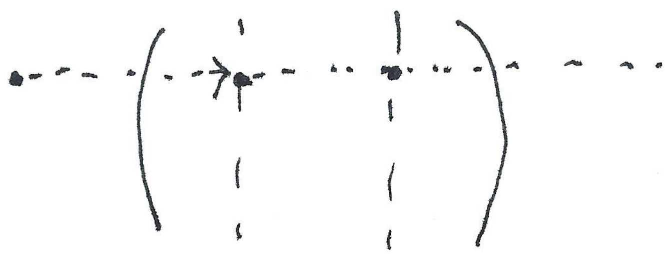
Focal points



focal planes

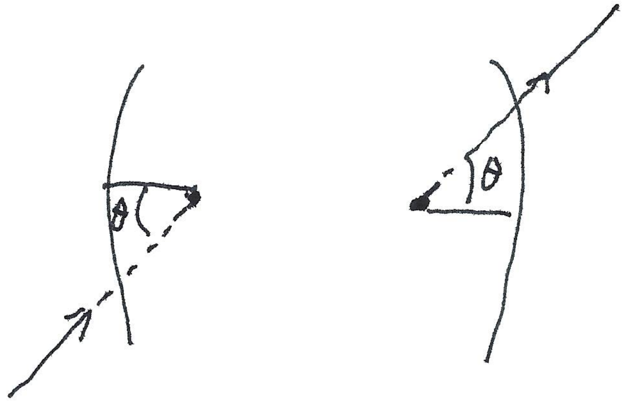
~~Principle~~ P

Principal points → principle planes

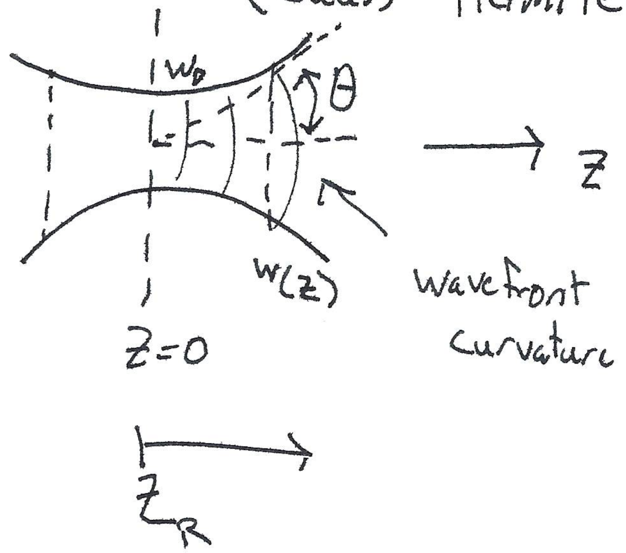


principle plane

Nodal points



Paraxial Gaussian Modes
(Gauss Hermite)



Paraxial divergence angle is small

Lowest paraxial "mode"

Guoy phase

~~Guoy phase~~

$$\vec{E}(x,y,z;\omega) = E_0 \hat{\epsilon}_\perp \frac{w_0}{w(z)} e^{\frac{-r^2}{w(z)^2}} e^{ikz + \frac{ikr^2}{2R(z)} - i\phi(z)}$$

↑ carrier phase ↑ wavefront curvature

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$$

$$z_R = \text{Rayleigh range} = \frac{\pi w_0^2 n}{\lambda}$$

n ↖ index of refraction

$R(z)$ = wavefront radius of curvature = ~~$z\left(1+\left(\frac{z}{z_R}\right)^2\right)$~~

$= z\left(1+\left(\frac{z_R}{z}\right)^2\right)$

Reaches maximum curvature at $z = z_R$

Guoy phase = $\arctan\left(\frac{z}{z_R}\right)$

Angular divergence $\theta \sim \arctan\left(\frac{w(z)}{z}\right) = \frac{\lambda}{\pi n w_0}$

$\theta w_0 \sim \frac{\lambda}{\pi n w_0} w_0 = \frac{\lambda}{\pi n}$ diffraction limited

$n \sin\theta = NA$ numerical aperture

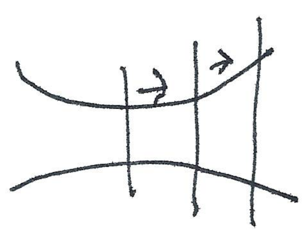
Beam Parameter Product (BPP)

$$\frac{\text{BPP of a real beam}}{\text{BPP of an ideal gaussian beam}} = M^2$$

"M-squared"

peak intensity = ~~$I(x,$~~ $I(x=0, y=0) = \frac{2P_0}{\pi w^2(z)}$ ← total power

P_0 = total power in beam = constant



Higher-order modes → Hermite polynomials in (x, y)

• Gaussian Mode

"paraxial wave equation"

$$\left(\nabla_\perp^2 + \left(\frac{\partial}{\partial z}\right)^2 + \frac{\omega^2}{c^2}\right) E = 0$$

$k_x, k_y \ll k_z$ $\quad E = E_{envelope} \cdot E_{carrier}$ $\quad E_{carrier} \sim e^{ikz}$

$\nabla_\perp^2$ *

$$\left(\nabla_\perp^2 - k^2 + 2ik \cdot \frac{\partial}{\partial z} + \frac{\omega^2}{c^2}\right) E_{envelope} \approx 0$$

$$2ik \frac{\partial}{\partial z}\psi + \nabla_\perp^2 \psi = 0$$

looks a lot like the Schrödinger equation

ψ is wave envelope

Send gaussian mode through an optical system

$$\begin{bmatrix} r \\ \theta \end{bmatrix}_f = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r \\ \theta \end{bmatrix}_i$$

↑ ray $\qquad$ ↑ ray tracing matrix

complex beam parameter $q(z) = z + iz_R$

$$q_f = \frac{Aq_i + B}{Cq_i + D}$$