Phys 110B 07 Dec 20

Ray Optics

ray equations:

$$\frac{d\vec{x}}{dt} = \frac{\partial \omega}{\partial \vec{k}}$$

$$\frac{d\vec{k}}{dt} = -\frac{\partial \omega}{\partial \vec{x}}$$

~~$\frac{\partial \omega}{\partial t} = \frac{\partial \omega}{\partial}$~~

$$\frac{d\omega}{dt} = \frac{\partial \omega}{\partial t}$$

Action conservation

"photon number"

"parallel Transport"

polarization

symmetries ⟷ conservation laws "Noether's Theorem"

if $\omega = \frac{c}{n(\vec{x},t)} \sqrt{\vec{k}\cdot\vec{k}}$

if $\frac{\partial n}{\partial x} = 0$ ⇒ k_x is conserved ie $\frac{dk_x}{dt} = 0$

if $\frac{\partial n}{\partial y} = 0$ $\quad \frac{dk_y}{dt} = 0$

if $\frac{\partial n}{\partial z} = 0$ $\quad \frac{dk_z}{dt} = 0$

if $\frac{\partial n}{\partial t} = 0$ $\quad \frac{d\omega}{dt} = 0$

if n is rotationally invariant about $\hat{z}$

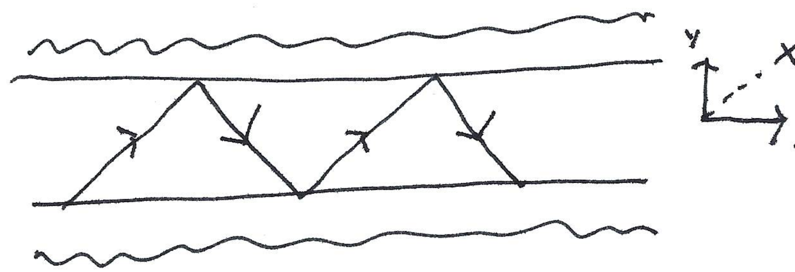
then $L_z = \hat{z} \cdot (\vec{r} \times \vec{k})$ is conserved

"skew invariant"

e.g. optical fiber

k_z is constant

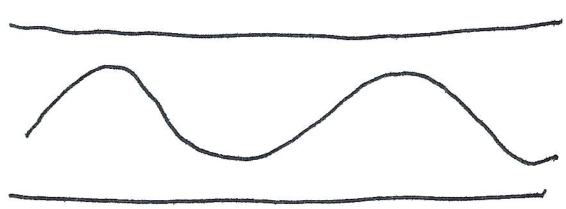
ω is constant



graded-index fiber

k_z is constant

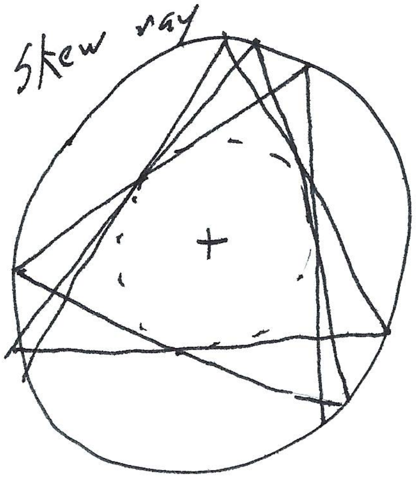
ω is constant

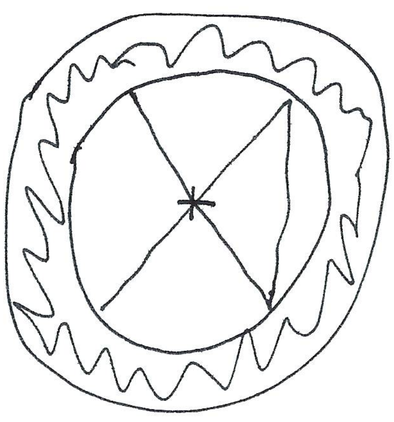


L_z is constant

head-on

skew ray





meridional rays cross the optic axis $L_z = 0$

skew rays do not intersect optic axis

$L_z \neq 0$

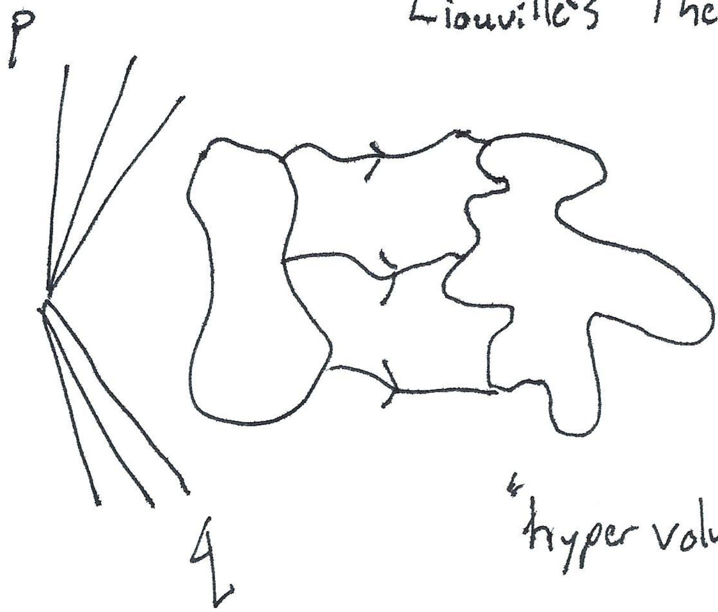
$|L_z| \propto$ smallest radius of approach to the optic axis

Dynamical invariants / conserved quantities

Noether's Theorem

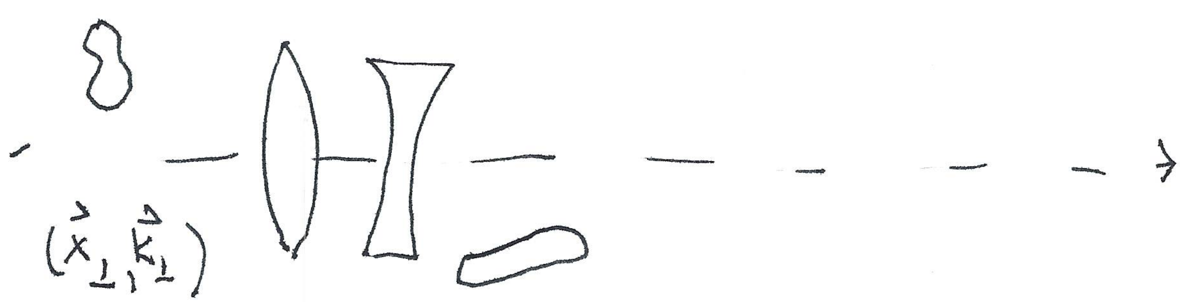
Phase space invariants

Liouville's Theorem

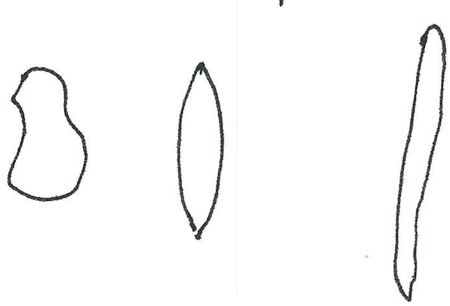


"hyper volume" is conserved

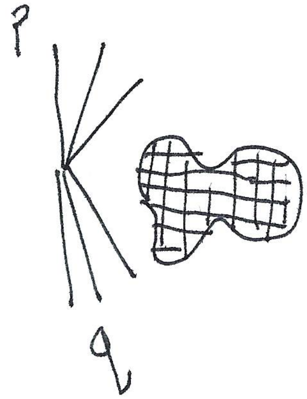
Liouville's Theorem in $(\vec{x}, \vec{k})$



Transverse phase-space "area" is "etendue"
extent



Classical phase space



phase space cells of size $(h)^{\text{power}}$

$$\#\text{ of quantum states} \;\frac{\text{phase space volume}}{h^{3N}}$$

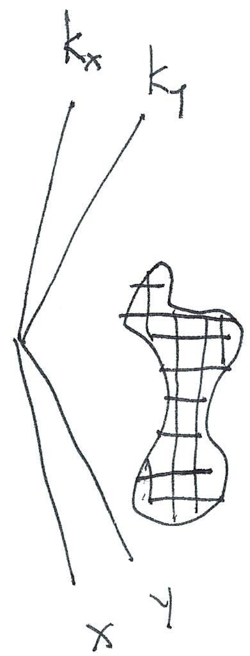
3 ← number of spatial dimensions; N ← # of particles

$$\sim \#\text{ of modes} \approx \frac{\text{hypervolume in } (\vec{x}, \vec{k})\text{ space}}{(2\pi)^3}$$

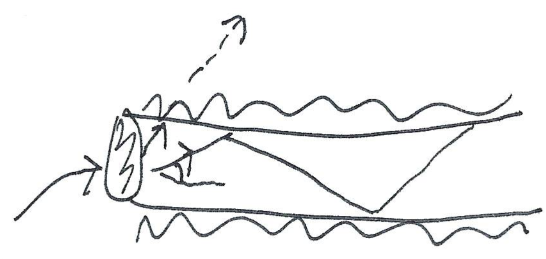
$$\#\text{ modes} = \#\text{ of transverse modes} \cdot \#\text{ of longitudinal modes}$$

$$\#\text{ of transverse modes} = \frac{\text{hyperarea in } (x, y, k_x, k_y)\text{ space}}{(2\pi)^2}$$

(~~hypervol~~)



$\approx \#$ of ⊥ modes



Asymptotically accurate ⟶ $\dfrac{\text{transverse "phasespace" area}}{(2\pi)^2}$ = Fiber parameter ≈ # of transverse modes

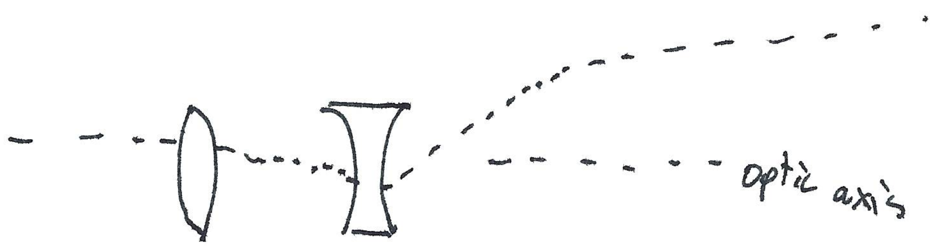
Typically geometric/ray optics limit is better when # of modes is big

$$\omega = \frac{c}{n}\sqrt{\vec{k}\cdot\vec{k}} \qquad n = n(\vec{x})$$

$$\frac{d\vec{x}}{dt} = \frac{\partial\omega}{\partial\vec{k}} = \frac{c}{n}\hat{k}$$

$$\frac{d\vec{k}}{dt} = \frac{-\partial\omega}{\partial\vec{x}} = \omega\frac{\vec{\nabla}n}{n}$$

$$\frac{d\omega}{dt} = \frac{\partial\omega}{\partial t} = 0$$



use z as evolution parameter

$$\frac{d\vec{x}}{dz} = \frac{\frac{d\vec{x}}{dt}}{\frac{dz}{dt}} = \frac{\frac{c}{n}\frac{\vec{k}}{|\vec{k}|}}{\frac{c}{n}\frac{k_z}{|\vec{k}|}} = \frac{\vec{k}}{k_z}$$

"paraxial" or near-axis approximation (small angles)

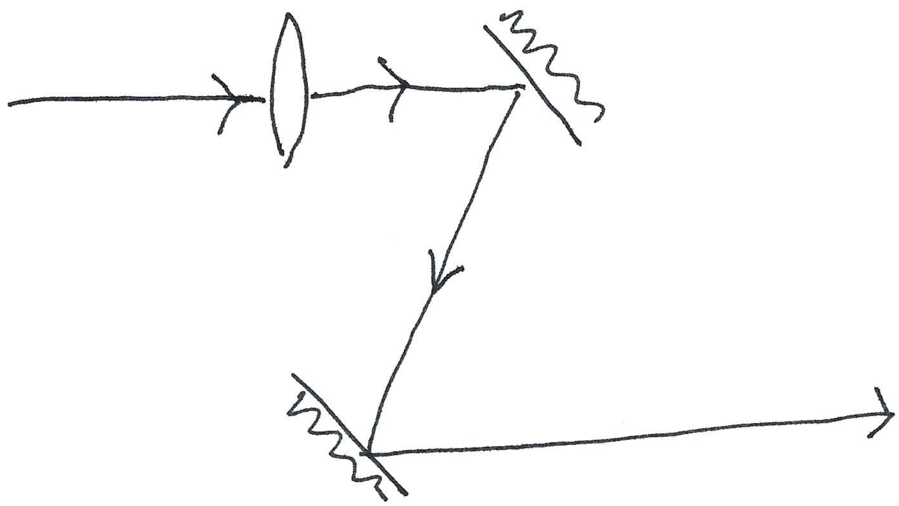
$$n\frac{dx}{dz} = n\frac{k_x}{k_z} \approx \frac{n}{k}k_x = \frac{c}{\omega}k_x$$

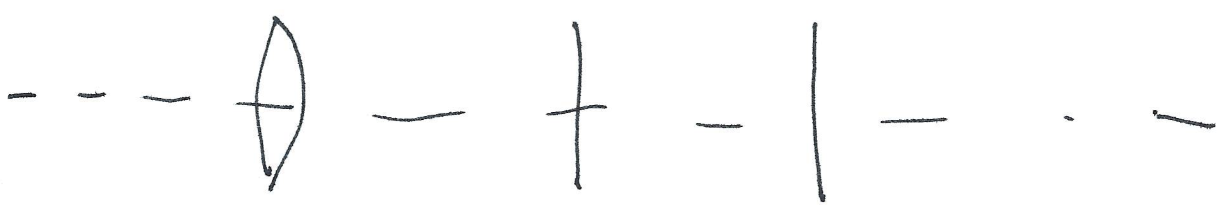
ω constant

Often axisymmetric assumption x and y are equivalent

Variables we need to track are $\begin{bmatrix} x \\ nx' \end{bmatrix}$ as a function of z along optic axis

"z" axis





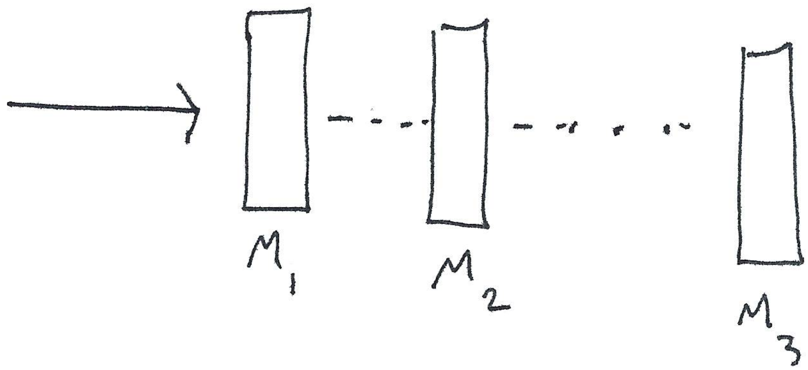
Ray-tracing

paraxial axisymmetric limits

↑ linearize in angles

↑ only x, x'

Ray-tracing ⟷ multiplying by ray-tracing matrices



$$\begin{bmatrix} x \\ nx' \end{bmatrix}\Bigg|_{z_f} = M_N \cdots M_3 M_2 M_1 \begin{bmatrix} x \\ nx' \end{bmatrix}\Bigg|_{z_i}$$

"drift" propagation in uniform medium

$$x_f' = x_i' \qquad n x_f' = n x_i'$$

$$x_f = x_i + \Delta z\, x_i'$$

$$\begin{bmatrix} x_f \\ n x_f' \end{bmatrix} = \begin{bmatrix} 1 & \frac{\Delta z}{n} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ n x_i' \end{bmatrix}$$

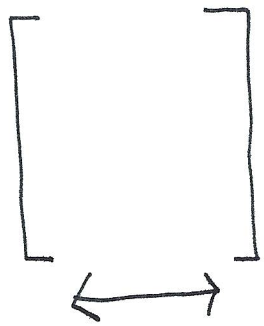
"Thin lens"

$$x_f = x_i$$

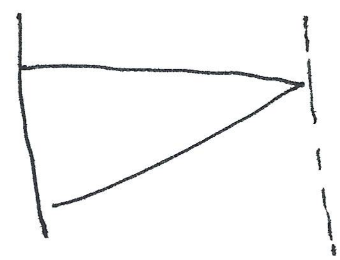
$$x_f' = x_i' + \frac{-n}{f} x_i$$

$$\begin{bmatrix} x_f \\ n x_i' \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{n}{F} & 1 \end{bmatrix}}_{\text{thin lens}} \begin{bmatrix} x_i \\ n x_i' \end{bmatrix}$$

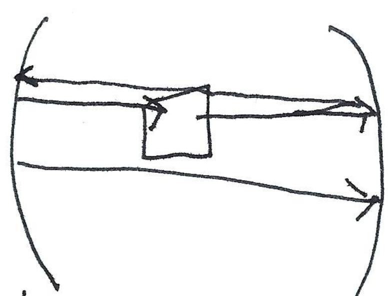
thick lens



e.g. stability of laser cavity



Flat mirrors are inefficient



Stability?

D drift
F reflect

Look at one round trip of a ray

$$\underset{\text{one round trip}}{M} = F_1 D F_2 D$$

unstable if one or more eigenvalues of M
$|\lambda| > 1$

$$M\vec{u} = \lambda\vec{u}$$

Liouville's Theorem $\Rightarrow \det M = 1$

$$\lambda_1 \lambda_2$$

stability requires $|\lambda_1| \leq 1 \quad |\lambda_2| \leq 1$

$$|\lambda_1| = |\lambda_2| = 1$$