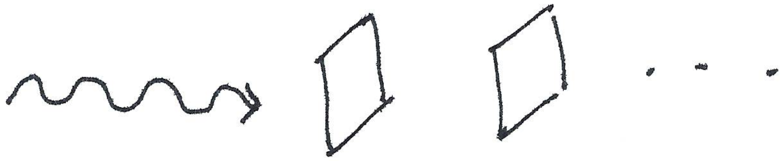
Phys 110B 02 Dec 20

Jones Calculus



J_1 J_2

$$\vec{\epsilon}_f = J_n \cdots J_2 J_1 \vec{\epsilon}_i$$

No polarization = statistically average to zero

1-point coherence tensor $\Gamma = \langle \vec{E} \vec{E}^\dagger \rangle$

outer product

collimated light beam 2x2 complex matrix

$$\vec{\epsilon}_f = J \vec{\epsilon}_i$$

$$\vec{\epsilon}_f^\dagger = \vec{\epsilon}_i^\dagger J^\dagger$$

$$\vec{\epsilon}_f \vec{\epsilon}_f^\dagger = J \vec{\epsilon}_i \vec{\epsilon}_i^\dagger J^\dagger$$

$$\langle \vec{\epsilon}_f \vec{\epsilon}_f^\dagger \rangle = J \langle \vec{\epsilon}_i \vec{\epsilon}_i^\dagger \rangle J^\dagger$$

$$\Gamma_f = J \Gamma_i J^\dagger$$

if $J = J_n J_{n-1} \cdots J_1$

$$J^\dagger = J_1^\dagger J_2^\dagger \cdots J_n^\dagger$$

$$\Gamma_f = J_n \cdots J_2 J_1 \Gamma_i J_1^\dagger J_2^\dagger \cdots J_n^\dagger$$

$$\Gamma = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_x^* E_y \rangle & \langle E_y E_y^* \rangle \end{bmatrix}$$

intensity $\propto |E_x|^2 + |E_y|^2 = Tr[\langle \vec{E}\vec{E}^\dagger \rangle] = Tr\,\Gamma$

trace of $\Gamma \propto$ intensity

$$\Gamma_f = J_n \cdots J_2 J_1 \Gamma_i J_1^\dagger J_2^\dagger \cdots J_n^\dagger$$

$$I_f \propto tr\,\Gamma_f = tr[J \Gamma_i J^\dagger] = tr[J^\dagger J \Gamma_i]$$

$$\Gamma = \langle \vec{E}\vec{E}^\dagger \rangle$$

$$\Gamma^\dagger = (\langle \vec{E}\vec{E}^\dagger \rangle)^\dagger = \langle \vec{E}^{\dagger\dagger} \vec{E}^\dagger \rangle = \Gamma$$

$\vec{\epsilon}^\dagger \Gamma \vec{\epsilon} \geq 0$ positive semidefinite

Γ is always unitarily diagonizable

$$\Gamma = \cancel{\lambda_1 \vec{\epsilon}_r \hat{\epsilon}_r^\dagger}\ \lambda_1 \hat{\epsilon}_1 \hat{\epsilon}_1^\dagger + \lambda_2 \hat{\epsilon}_2 \hat{\epsilon}_2^\dagger$$

eigen vectors to be orthonormal

$$\hat{\epsilon}_1^\dagger \hat{\epsilon}_1 = \hat{\epsilon}_2^\dagger \hat{\epsilon}_2 = 1$$

$$\hat{\epsilon}_1^\dagger \hat{\epsilon}_2 = \hat{\epsilon}_2^\dagger \hat{\epsilon}_1 = 0$$

$$\Gamma = \lambda_1 \hat{\epsilon}_1 \hat{\epsilon}_1^\dagger + \lambda_2 \hat{\epsilon}_2 \hat{\epsilon}_2^\dagger$$

any polarization state = mixture of two "eigen" polarizations

Totally unpolarized light

$\Gamma \propto \mathbb{1}$ or equal amounts of $\hat{\epsilon}_1 \hat{\epsilon}_1^\dagger + \hat{\epsilon}_2 \hat{\epsilon}_2^\dagger$

$\vec{L}\vec{L}^\dagger + \vec{R}\vec{R}^\dagger$

$\vec{H}\vec{H}^\dagger + \vec{V}\vec{V}^\dagger$ etc.

$$\Gamma = \lambda_1 \vec{\epsilon}_1 \vec{\epsilon}_1^\dagger + \lambda_2 \vec{\epsilon}_2 \vec{\epsilon}_2^\dagger$$

$$\lambda_1 \geq \lambda_2 \geq 0$$

$\cancel{\Gamma = (\lambda_1 - \lambda_2)}$

$$\Gamma = \lambda_2 (\hat{\epsilon}_1 \hat{\epsilon}_1^\dagger + \hat{\epsilon}_2 \hat{\epsilon}_2^\dagger) + (\lambda_1 - \lambda_2) \hat{\epsilon}_1 \hat{\epsilon}_1^\dagger$$

$$= \lambda_2 \mathbb{1} + (\lambda_1 - \lambda_2) \hat{\epsilon}_1 \hat{\epsilon}_1^\dagger$$

any polarization

mixture of totally polarized

totally polarized along $\hat{\epsilon}_1$

degree of polarization $= \dfrac{\text{total polarized intensity}}{\text{total intensity}} = \dfrac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$

$$0 \le \text{dop} \le 1$$

0: totally polarized

1: totally polarized

Can express dop in terms of $\mathrm{tr}\,\Gamma$ and $\mathrm{tr}\,\Gamma^2$

Similarities between Γ and density operator ρ in QM

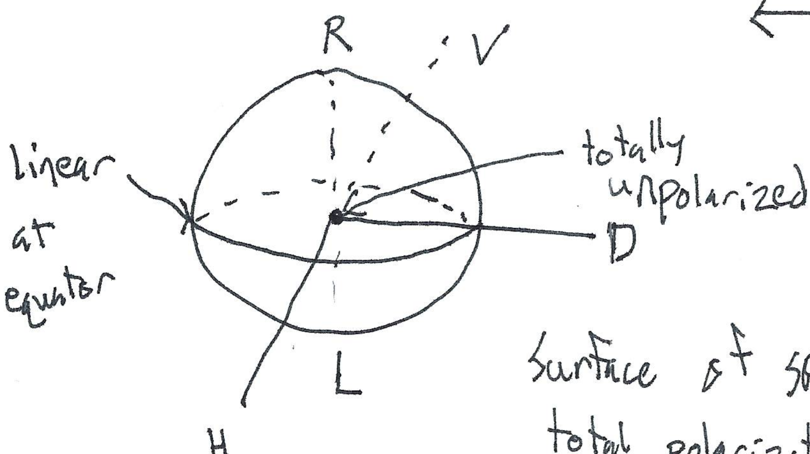
$\rho = \sum w_i |\psi_i\rangle\langle\psi_i|$ $\quad \langle A \rangle = \mathrm{Tr}[\rho A]$

EM polarization $\longleftrightarrow$ spin of photons

$\Gamma \propto \rho$ for "photon spin"

Classical EM

Poincaré sphere $\longleftrightarrow$ "Bloch Sphere"

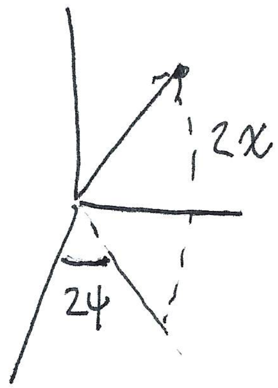


Surface of sphere

total polarization / pure polarization

inside: partial polarization

"mixed" polarization



"coordinates" of Poincaré sphere are "Stokes Parameters"

S_0 I $\langle|E_x|^2\rangle + \langle|E_y|^2\rangle = \langle|E_R|^2\rangle + \langle|E_L|^2\rangle$

S_1 Q $\langle|E_x|^2\rangle - \langle|E_y|^2\rangle = 2\mathrm{Re}[\langle E_R^* E_L\rangle]$

S_2 U $2\mathrm{Re}(\langle E_x E_y^*\rangle) = -2\,\mathrm{Im}[\langle E_R^* E_L\rangle]$

S_3 V $-2\mathrm{Im}(\langle E_x E_y^*\rangle) = \langle|E_R|^2\rangle - \langle|E_L|^2\rangle$

$\Gamma \longleftrightarrow S_0, S_1, S_2, S_3$ $\cancel{I_p \cos 2\chi \cos 2\psi}$

$\Gamma^\dagger = \Gamma \Rightarrow$ 4 real parameters

$I_p \cos 2\chi \cos 2\psi = S_1$

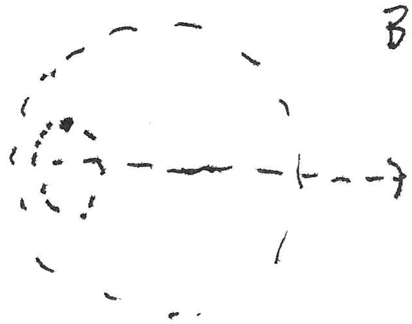
$I_p \cos 2\chi \sin 2\psi = S_2$

$I_p \sin 2\chi = S_3$

$I_p^2 = Q^2 + U^2 + V^2 \le I^2$

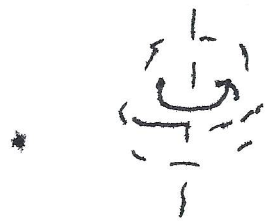
$S_1^2 + S_2^2 + S_3^2$

$\mathrm{dop} = \dfrac{I_p}{I}$ $\dfrac{|Q + iU|}{I}$ = degree of linear polarization



Birefringence → rotate polarization on Poincaré sphere

optical activity / circular birefringence



Ray Optics
Geometric Optics
Eikonal Approximation
WKB Approximation

Short wavelength asymptotics
Local plane wave Approximation

$\lambda \ll$ length scales of variation of optical properties of ~~thn~~ the medium

index of refraction $n(\vec{x})$

$$L^{-1} \sim \frac{|\vec{\nabla} n|}{|n|}$$

$\lambda \ll L$

Maxwell $\rightarrow$ Approximations $\downarrow$ eikonal / WKB

variational / Lagrangian
Fermat's Principle

Hamiltonian optics

ray-tracing or ray optics