Phys 110B Dis 28Oct20

Griffiths 10.11

Long (∞) ~~stragi~~ straight wire carrying current $I(t)$

b) Find $\vec{E}$ and $\vec{B}$ fields when $I(t) = q\,\delta(t)$

Azimuthal symmetric
Translationally symmetric

Choose current to flow along $\hat{z}$ axis

$J(x', y', z', t')$

$\vec{A}$ In Lorenz-Lorentz gauge

$$\vec{A}(\vec{x}, t) = \frac{\mu_0}{4\pi}\int dx'dy'dz' \frac{\left[q\,\delta(t')\,\delta(x')\,\delta(y')\right]_{ret}}{\left[\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}\right]_{ret}}\,\hat{z}$$

$$= \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty} dz' \,\frac{q\,\delta(t - R/c)}{R}\,\hat{z} \qquad R = \sqrt{x^2+y^2+(z-z')^2}$$

shift to cylindrical coordinates $s = \sqrt{x^2+y^2},\ \phi,\ z$

for the moment choose $z = 0$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty} dz'\,\hat{z}\,q\,\frac{\delta(t-\frac{R}{c})}{R} \qquad R = \sqrt{s^2+z'^2}$$

$$= 2\frac{\mu_0}{4\pi}\int_0^{\infty} q\,\frac{\delta(t-\frac{R}{c})}{R}\,\hat{z}\,dz' \qquad z' = \sqrt{R^2-s^2} \Rightarrow dz' = \frac{1}{2}\frac{2R\,dR}{\sqrt{R^2-s^2}}$$

$$= \frac{R\,dR}{\sqrt{R^2-s^2}}$$

$$= \frac{\mu_0 q}{2\pi}\hat{z}\int_s^{\infty} dR\,\frac{R\,\delta(t-R/c)}{\sqrt{R^2-s^2}\;R}$$

$$= \frac{\mu_0 q}{2\pi}\, z \int_s^{\infty} \frac{\cancel{R}dR}{\sqrt{R^2-s^2}}\,\delta(R-ct'')\,c$$

$$\vec{A} = \frac{\mu_0 q c}{2\pi}\hat{z}\begin{cases} \dfrac{1}{\sqrt{c^2t^2-s^2}} & \text{if } ct > s \\ 0 & \text{if } ct < s \end{cases}$$

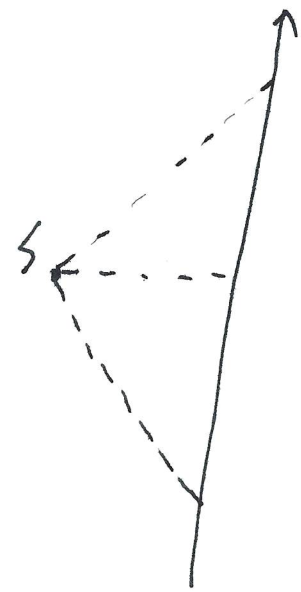
$$\vec{E} = -\frac{\partial}{\partial t}\vec{A} = \frac{\mu_0 q c}{2\pi}\left(-\frac{1}{2}\right)\frac{2c^2 t}{(c^2t^2-s^2)^{3/2}}\hat{z}$$

$$= \frac{\mu_0 q c^3 t\,\hat{z}}{2\pi(c^2t^2-s^2)^{3/2}} \quad \text{if } ct \geq s$$

or $\vec{E} = 0$ otherwise

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{-\mu_0 q c s\,\hat{\phi}}{2\pi(c^2t^2-s^2)^{3/2}} \quad \text{if } ct \geq s$$

or $\vec{B} = 0$ otherwise



receives signal for all t after the delay

Griffiths 10.17

Hyperbolic motion $x(t) = \sqrt{b^2 + (ct)^2}$

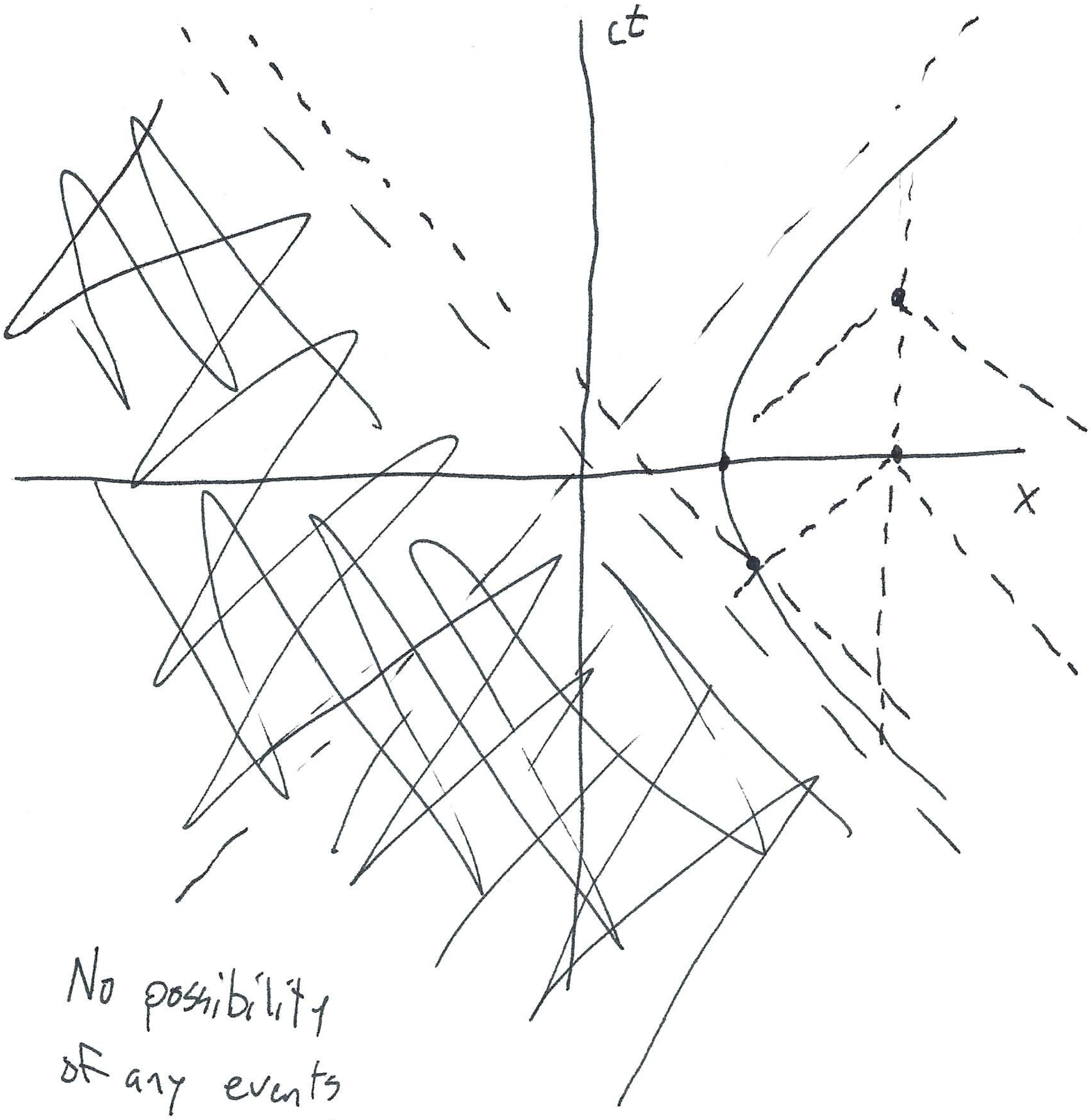
$-\infty < t < \infty$

Sketch a spacetime diagram showing a) the particle trajectories

b) some forward light cones from the particle

c) all spacetime points for which the particle remains invisible

d) a given x, show times the particle becomes visible.



No possibility of any events over here to be aware of our hyperbola

acceleration Horizon

charge in hyperbolic motion may not be radiating