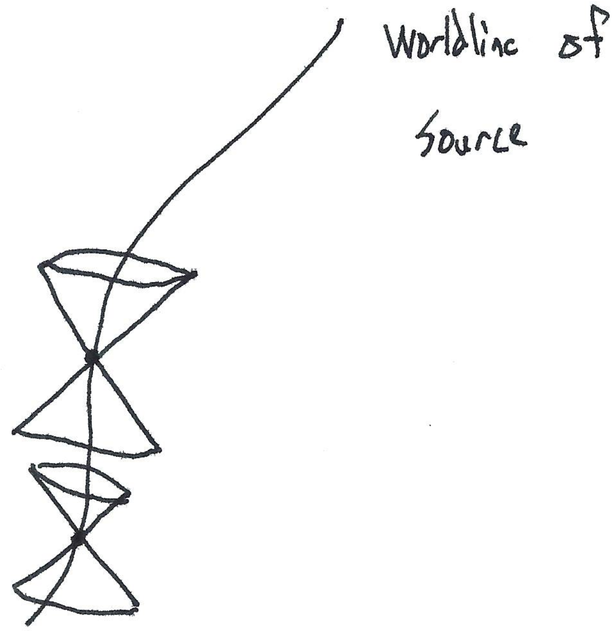
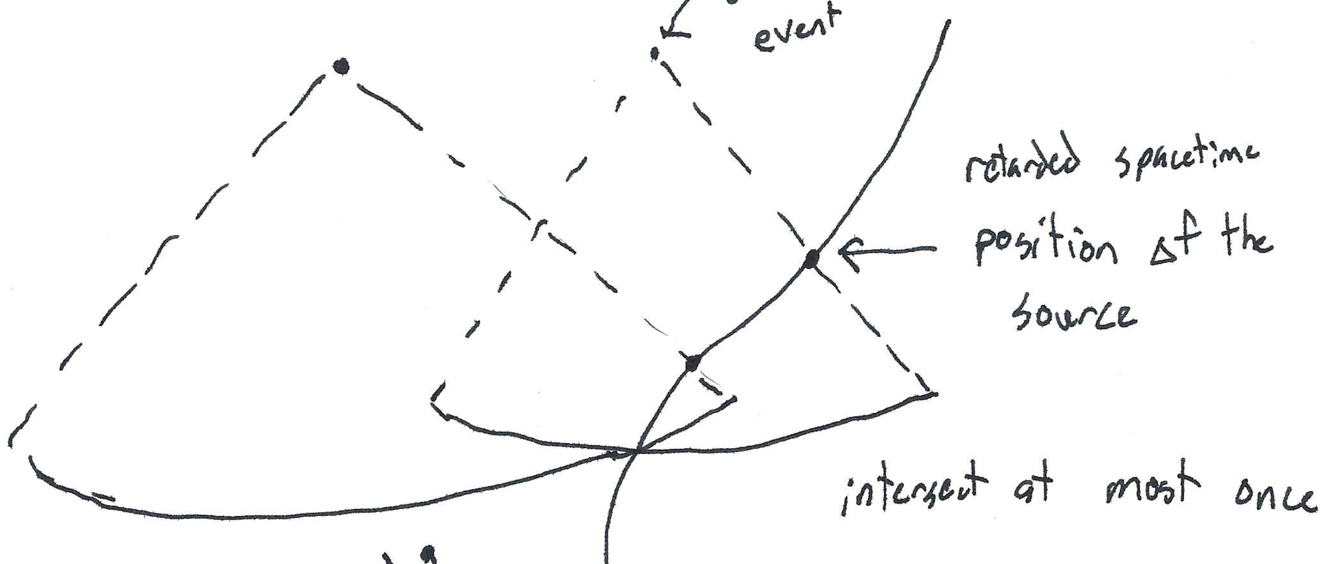
Phys 110B DIS 04Nov20





Griffiths 11.1

verify that the retarded potentials of an oscillatory dipole satisfy the Lorentz-Lorenz gauge exactly eqs 11.12 and 11.17

$$\Phi(r,\theta,t) = \frac{p_0 \cos\theta}{4\pi\epsilon_0 r}\left(-\frac{\omega}{c}\sin(\omega(t-r/c)) + \frac{1}{r}\cos(\omega(t-\frac{r}{c}))\right)$$

$$\vec{A}(r,\theta,t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\sin(\omega(t-\frac{r}{c}))\hat{z} \qquad \hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

$$\vec{\nabla}\cdot\vec{A} = -\frac{\mu_0 p_0 \omega}{4\pi}\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r\sin(\omega(t-\frac{r}{c}))\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(-\sin^2\theta\frac{1}{r}\sin(\omega(t-\frac{r}{c}))\right)\right)$$

$$= \frac{-\mu_0 p_0 \omega}{4\pi}\left(\frac{1}{r^2}\left(\cos\theta\sin(\omega(t-\frac{r}{c})) - \frac{\omega r}{c}\cos(\omega(t-\frac{r}{c}))\cos\theta\right)\right.$$

$$\left. + \frac{1}{r\sin\theta}\left(-2\sin\theta\cos\theta\frac{1}{r}\sin(\omega(t-\frac{r}{c}))\right)\right)$$

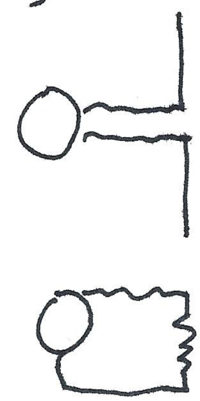
$$= \mu_0\epsilon_0\left(\frac{p_0\omega}{4\pi\epsilon_0}\left(\frac{1}{r^2}\sin(\omega(t-\frac{r}{c})) + \frac{\omega}{rc}\cos(\omega(t-\frac{r}{c}))\right)\right)$$

$$\frac{\partial\Phi}{\partial t} = -\frac{p_0\cos\theta}{4\pi\epsilon_0 r}\left(\frac{\omega}{r}\sin(\omega(t-\frac{r}{c})) + \frac{\omega^2}{c}\cos(\omega(t-\frac{r}{c}))\right)$$

$$\vec{\nabla}\cdot\vec{A} = -\epsilon_0\mu_0\frac{\partial\Phi}{\partial t} \quad \checkmark$$

Griffiths 11.3

Find the radiation resistance of an oscillating dipole



Show that $R = 790 \left(\frac{d}{\lambda}\right)^2 \Omega$

$$R = \frac{2}{3}\pi\mu_0 c\left(\frac{d}{\lambda}\right)^2$$

$$I = -q_0\omega \sin\omega t$$

$$P = I^2 R = q_0^2\omega^2 \sin^2\omega t \, R$$

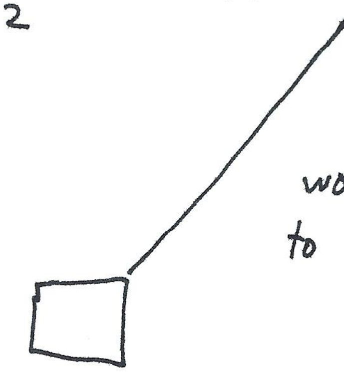
$$\langle P \rangle = \frac{1}{2} q_0^2 \omega^2 R = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$$

$$R = \frac{\mu_0 p_0^2 \omega^2}{6\pi q_0^2 c} = \frac{\mu_0 d^2 \omega^2}{6\pi c}$$ since $\frac{p_0^3}{q_0^2} = d^2$

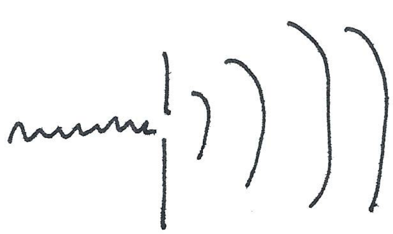
and $\omega = \frac{2\pi c}{\lambda}$

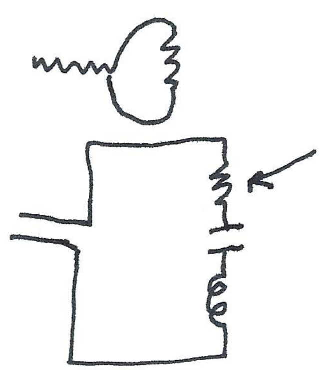
$$R = \frac{4\pi^2 \mu_0 d^2 c^2}{6\pi c \lambda^2} = \frac{2}{3}\pi\mu_0 c\left(\frac{d}{\lambda}\right)^2$$

$$\frac{R_{rad}}{R_{wire}} \sim 10^{-2} \ll 1$$



antenna won't be able to ignore R_{rad}





only looking at the resistor