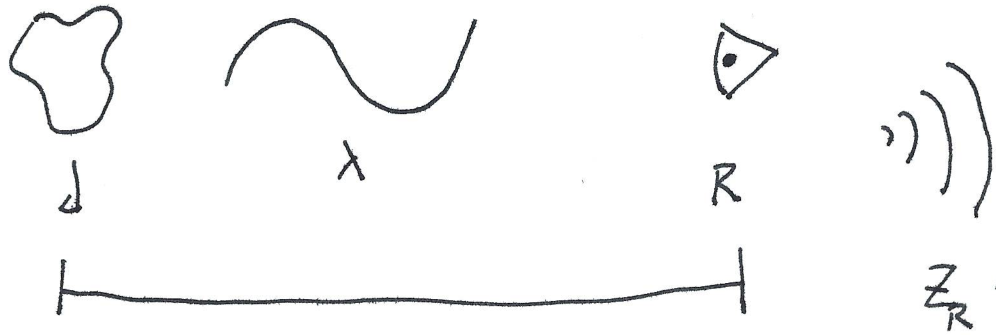
Phys 110B 30Oct20



$$Z_R \sim \frac{d^2}{\lambda}$$

Hertzian Regime $d \ll \lambda$

Near-zone $\quad d \lesssim R \ll \lambda$

Quasi-static zone

Intermediate zone $\quad d \ll \lambda \sim R$

Induction zone

far zone $\quad d \ll \lambda \ll R$

radiation zone

Diffraction Theory

Fresnel number $\quad F \sim \dfrac{Z_R}{R}$

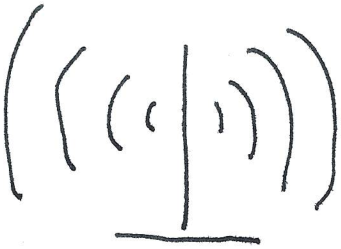
$F \gtrsim 1$ Fresnel Diffraction

"near diffraction field"

$F \ll 1$ Fraunhofer Diffraction $\quad R > d$

"far-field" diffraction

Antenna Theory



"true" Far field $R >> Z_R$

radiative near field

reactive near field $R \lesssim \lambda$

Hertzian Multipolar Radiation $d << \lambda$

(quasi-static) near zone $d < R << \lambda$

$kd < Kr << 1$

Lorentz-Lorenz gauge

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int \frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}\vec{J}(\vec{x}) \qquad \omega = ck$$

in Fourier space

in near field $kr << 1 \Rightarrow e^{ik|\vec{x}-\vec{x}'|} \approx 1$

$\vec{A}(\vec{x})$ "looks" like the static case

Able to expand $\vec{A}(\vec{x})$ in quasistatic multipole contribution

$$\frac{1}{|\vec{x}-\vec{x}'|} = 2\pi \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}} Y^*_{lm}(\theta',\phi') Y_{lm}(\theta,\phi)$$

observation $\vec{x} = (r,\theta,\phi)$ $\qquad r_< = \mathrm{Min}[r,r']$

source $\vec{x}' = (r',\theta',\phi')$ $\qquad r_> = \mathrm{Max}[r,r']$

$kd < |\vec{x}| << 1$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \sum_l \sum_m \frac{4\pi}{2l+1} \frac{Y_{lm}(\theta, \phi)}{r^{l+1}} \int \vec{J}(\vec{x}'; \omega) \, r'^l \, Y^*_{lm}(\theta', \phi') \, d^3\vec{x}'$$

(radiation) far zone $d << \lambda << R$

$$|\vec{x} - \vec{x}'| = \left((\vec{x} - \vec{x}') \cdot (\vec{x} - \vec{x}') \right)^{1/2}$$ Taylor expand treating $\vec{x}'$ as small

$$\simeq |\vec{x}| - \hat{n} \cdot \vec{x}' + \ldots$$

r $\quad \hat{n} = \frac{\vec{x}}{|\vec{x}|}$ unit vector in direction of observation point

$$\vec{A}(\vec{x}; \omega) = \frac{\mu_0}{4\pi} \int \vec{J}(\vec{x}') \frac{e^{ik|\vec{x} - \vec{x}'|}}{|\vec{x} - \vec{x}'|} d^3\vec{x}'$$

can't pull out as it is an exponential

can pull out as it is just a magnitude

$$\cong \frac{\mu_0}{4\pi} \frac{1}{r} e^{ikr} \int d^3\vec{x}' \, \vec{J}(\vec{x}') \, e^{-ik\hat{n} \cdot \vec{x}'}$$

$$= \frac{\mu_0}{4\pi} \underset{\text{spherical wave}}{\frac{e^{ikr}}{r}} \underbrace{\int d^3\vec{x}' \, \vec{J}(\vec{x}'; \omega) \, e^{-ik\hat{n} \cdot \vec{x}'}}_{\text{spatial Fourier Transform of Source current density}}$$

$$\frac{1}{\sqrt{2\pi}} \int d^3\vec{x}' \, \vec{J}(\vec{x}; \omega) e^{i\vec{k} \cdot \vec{x}'} = J(\vec{k}, \omega)$$

$$\vec{k} = k\hat{n} \qquad k = \frac{\omega}{c} \qquad \hat{n} = \frac{\vec{x}}{r}$$

Because $d \ll \lambda$, Taylor expand $e^{-ik\hat{n}\cdot\vec{x}'}$ in powers of k

$$\vec{A}(\vec{x};\omega) \cong \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\sum_{n=0}^{\infty}\frac{(-ik)^n}{n!}\int \vec{J}(\vec{x}')(\hat{n}\cdot\vec{x}')^n d^3\vec{x}'$$

successive terms grow smaller by powers of $kd \ll 1$

(Induction) Intermediate zone

Neither Approximation works

Can expand the Green function $\dfrac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$

$$\vec{A}(\vec{x};\omega) = \mu_0 ik \sum_{l}\sum_{m} Y_{lm}(\theta,\phi)\, h_l^{(1)}(kr)\int \vec{J}(\vec{x}')\, j_l(kr)\, Y_{lm}^*(\theta',\phi')\, d^3\vec{x}'$$

Spherical Bessel Functions

when $d \ll \lambda$, $h_l^{(1)}(kr)$ and $j_l(kr)$ can be expanded

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\sum_{l}\sum_{m}\frac{4\pi}{2l+1}Y_{lm}(\theta,\phi)\frac{e^{ikr}}{r^{l+1}}\left(1 + a_1(ikr) + a_2(ikr)^2 + \dots\right)\int d^3\vec{x}'\,\vec{J}(\vec{x}')\, r'^{l}\, Y_{lm}^*(\theta',\phi')$$

Why only looking at vector potential?

$$\Phi(x,t) = \int d^3\vec{x}'\int dt'\,\frac{\rho(\vec{x}',t')}{|\vec{x}-\vec{x}'|}\,\delta\!\left(t' + \frac{|\vec{x}-\vec{x}'|}{c} - t\right)$$

$|\vec{x}-\vec{x}'| \to r$ monopole contribution

$\Phi(\vec{x},t) \simeq \dfrac{q(t' = t - \frac{r}{c})}{r}$ but if the source is localized then $q(t')$ is fixed due to charge conservation

$q(t)$ is constant in time

monopole has only $\omega = 0$ contributions

those are static ($\frac{1}{r^2}$ fields)

Radiation has matched $\vec{E}, \vec{B}$ fields

$\vec{A}$ has all the information about radiation fields

Far zone of a Hertzian Radiator

leading order term

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int \vec{J}(\vec{x}')\, d^3\vec{x}'$$

(0th order taylor series term in far zone)

($l=0$ term in the induction zone expansion)

express in terms of electric dipole moment $\vec{p}$ of $\vec{J}(\vec{x}')$