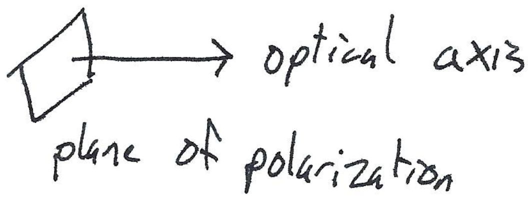
Phys 110B 30Nov20

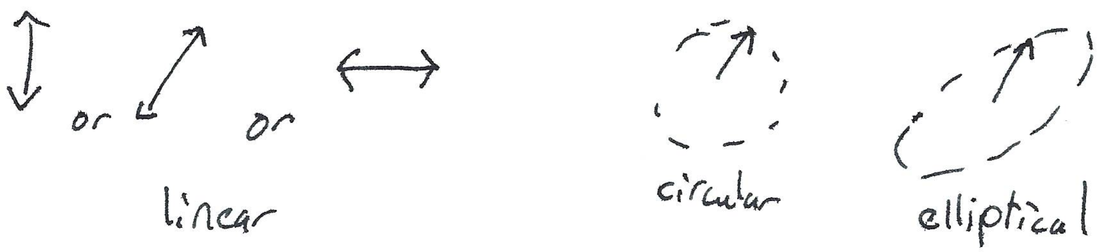
Polarization "Jones Calculus"

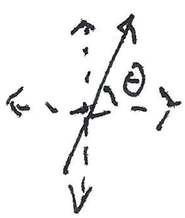
well-collimated beam of EM radiation

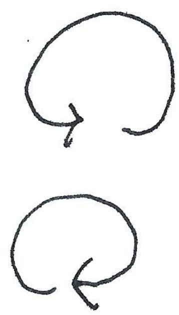



polarization → direction of $\vec{E}$ field

polarization states ↔ 2D complex vectors



Jones vector complex 2D polarization vector in some basis

Horizontal $\leftrightarrow$ $\vec{H}$ $|H\rangle$

vertical $\updownarrow$ $\vec{V}$ $|V\rangle$

diagonal $\swarrow\!\!\!\nearrow$ $\vec{D}$ $|D\rangle$

anti-diagonal $\nwarrow\!\!\!\searrow$ $\vec{A}$ $|A\rangle$

left circular $\circlearrowleft$ $\vec{L}$ $|L\rangle$

right circular $\circlearrowright$ $\vec{R}$ $|R\rangle$

Orthonormal basis makes life easier

$$\vec{H}^\dagger\vec{H}=1 \quad \vec{V}^\dagger\vec{V}=1 \quad \vec{H}^\dagger\vec{V}=0$$

$$\langle H|H\rangle=1 \quad \langle V|V\rangle=1 \quad \langle H|V\rangle=0$$

In this $\vec{H},\vec{V}$ basis

$$\vec{H}\cong\begin{bmatrix}1\\0\end{bmatrix} \quad \vec{V}\cong\begin{bmatrix}0\\1\end{bmatrix}$$

$$\vec{E}=\begin{bmatrix}\alpha\\\beta\end{bmatrix}=\alpha\begin{bmatrix}1\\0\end{bmatrix}+\beta\begin{bmatrix}0\\1\end{bmatrix}$$

$$\langle\vec{S}\rangle=\sqrt{\frac{\epsilon}{\mu}}\,\frac{c}{8\pi}\left(\langle E_x^\dagger E_x\rangle+\langle E_y^\dagger E_y\rangle\right)\propto|\alpha|^2+|\beta|^2\propto\vec{E}^\dagger\vec{E}$$

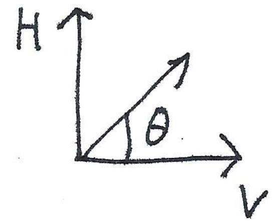
In $\vec{H}, \vec{V}$ basis

$\vec{D} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ $\quad$ $\vec{A} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$

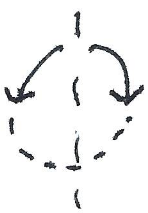
georgi waves convention for $\vec{L}, \vec{R}$

$\vec{L} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$ $\quad$ $\vec{R} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$

$\hat{\epsilon}_\theta = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$ $\quad$ $\hat{\epsilon}_{\theta+\frac{\pi}{2}} = \begin{bmatrix}\sin\theta\\ -\cos\theta\end{bmatrix}$

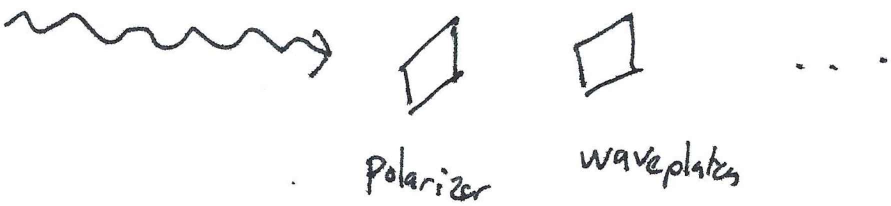



in $\vec{L}, \vec{R}$ basis



$$\frac{\vec{L}+\vec{R}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = \vec{H}$$

$$\frac{\vec{L}-\vec{R}}{\sqrt{2}\,i} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{i}\begin{bmatrix}0\\2i\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} = \vec{V}$$




"device" in the beam line $\longleftrightarrow$ 2x2 complex matrix

<u>Linear Polarizer</u> (idealized case)

$$P_H = \begin{bmatrix}1&0\\0&0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1&0\end{bmatrix} = \vec{H}\vec{H}^\dagger$$

horizontal

$$P_V = \begin{bmatrix}0&0\\0&1\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0&1\end{bmatrix} = \vec{V}\vec{V}^\dagger$$

$$P_\theta = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$P_\theta$ = Hermitian Projections

$$P_\theta^\dagger = P_\theta \qquad P_\theta P_\theta = P_\theta \quad \text{(idempotent)}$$

$$(\hat u \hat u^\dagger) = \hat u^{\dagger\dagger} \hat u^\dagger = \hat u \hat u^\dagger$$

$$(\hat u \hat u^\dagger)(\hat u \hat u^\dagger) = \hat u(\hat u^\dagger \hat u)\hat u^\dagger = \hat u \hat u^\dagger$$

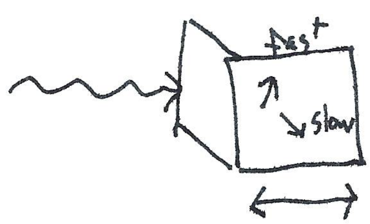
$$\hat u \cdot \hat u = 1$$

<u>filter</u>

$$F = f\,\mathbb{1} \qquad 0 \le f \le 1$$

identity

<u>Waveplates</u> birefrigance




Quarter-wave plate

$$Q_H = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}_{H,V} = P_H + i P_V$$

fast axis

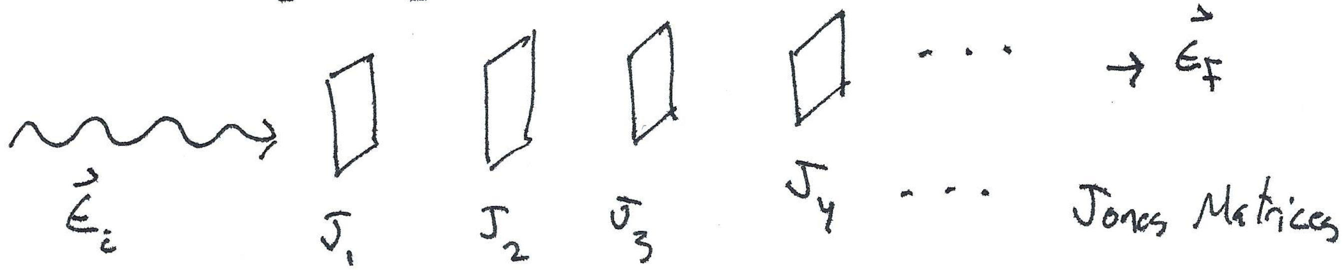
$$Q_\theta = P_\theta + i P_{\theta + \frac{\pi}{2}}$$

fast axis

Half waveplate

$$\mathcal{H}_\theta = \mathcal{Q}_\theta \mathcal{Q}_\theta$$

$$\mathcal{H}_H = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$




$$\vec{E}_f = J_n \cdots J_4 J_3 J_2 J_1 \vec{E}_i$$

$$\vec{E}_f^{\dagger} \vec{E}_f \propto \langle \vec{S} \rangle$$