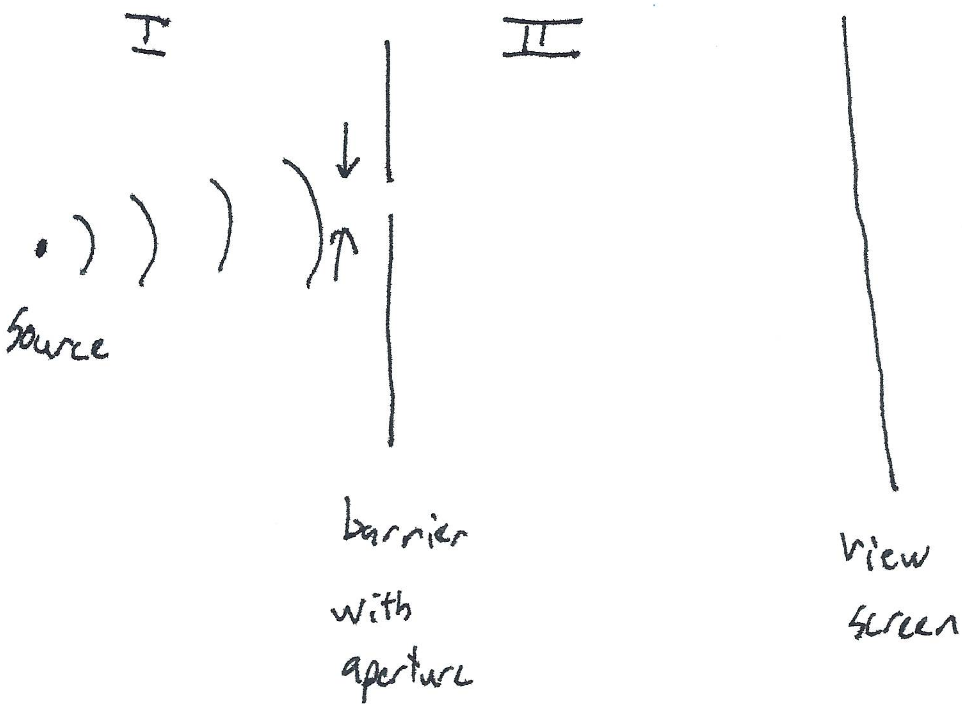
Phys 110B 20Nov20

Scattering, Interference, and Diffraction

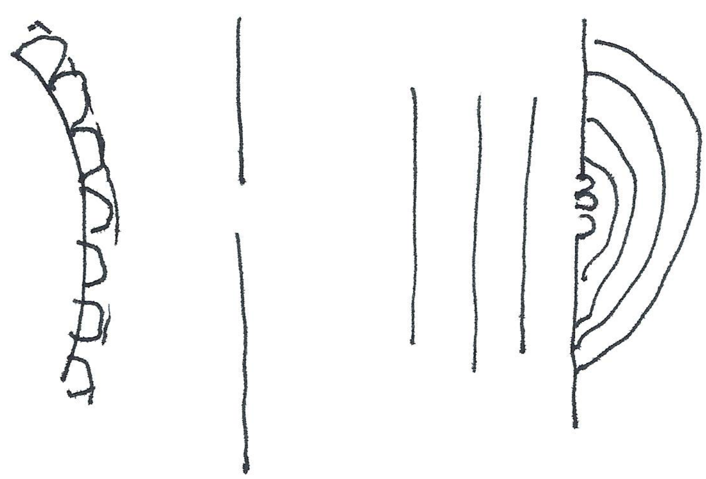


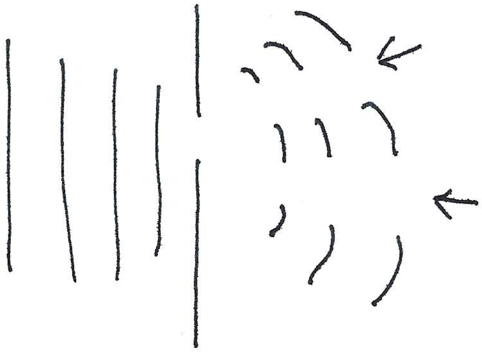
Diffraction from aperture or obstacle

typically $d \gg \lambda$ $\quad \theta \sim \frac{\lambda}{d} \ll 1$

Huygens
Young
Fresnel
Kirchoff
Rayleigh
Sommerfeld
Stratton-Chu
etc

Huygens Principle





Waves are made of small wave packets
Not really true physically but leads to correct result

Kirchoff Formula

- Scalar diffraction theory

$$\psi(\vec{x},t) \to \psi(\vec{x};\omega) \quad (\nabla^2 + k^2)\psi = 0$$

Green's Second Identity

$$\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) d^3\vec{x} = \int_{\partial V} (\phi \vec{\nabla}\psi - \psi \vec{\nabla}\phi) \cdot \hat{n}\, da$$

$\psi = \psi \quad \phi = G(\vec{x},\vec{x}') =$ green function

$$(\nabla^2 + k^2) G = -\delta(\vec{x} - \vec{x}')$$

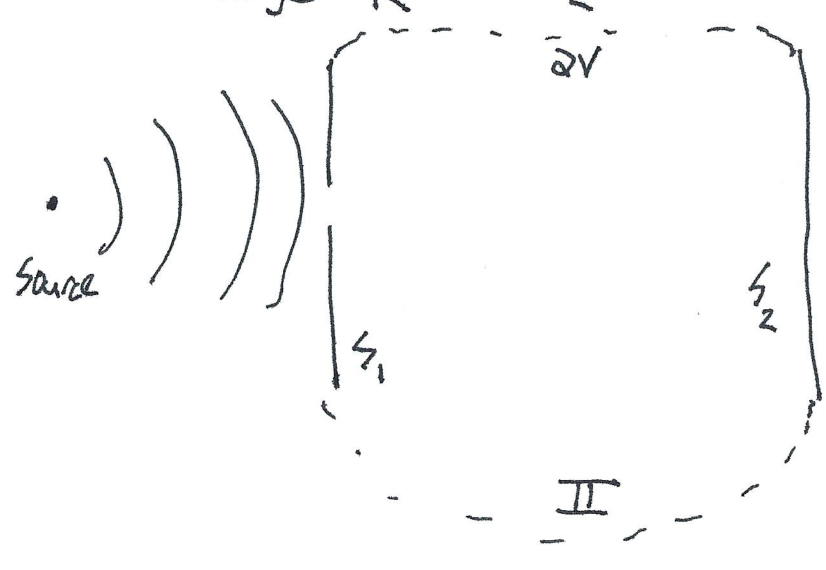
Green's Identity →

$$\psi(x) = \iint_{\partial V} da' [\psi \hat{n}' \cdot \vec{\nabla}' G - G \hat{n}' \cdot \vec{\nabla}' \psi)$$

when $x \in V$ ↑ inward normal

$$G = \frac{e^{ikR}}{4\pi R} \quad \vec{R} = \vec{X} - \vec{X}'$$ (free space Green Function)

$$\psi(x) = \frac{-1}{4\pi}\iint \frac{e^{ikR}}{R}\,\hat{n}'\cdot\left[\vec{\nabla}'\psi + ik\left(1+\frac{i}{kR}\right)\hat{R}\psi\right]da'$$



take $S_2 \to \infty$ $\qquad\qquad \partial V = S_1 + S_2$

integral will vanish on S_2 as $S_2 \to \infty$

ψ, $\hat{n}\cdot\vec{\nabla}\psi$ vanish where S_1 is a barrier

ψ, $\hat{n}\cdot\vec{\nabla}\psi$ assume the same values they would have in the absence of the barrier in any aperture (Approximation)

$$\psi(x) = -\frac{1}{4\pi}\int_{S_1} \frac{e^{ikR}}{R}\,\hat{n}'\cdot\left[\vec{\nabla}'\psi + ik\left(1+\frac{i}{kR}\right)\right]\frac{\vec{R}}{R}\,da'$$

in region II

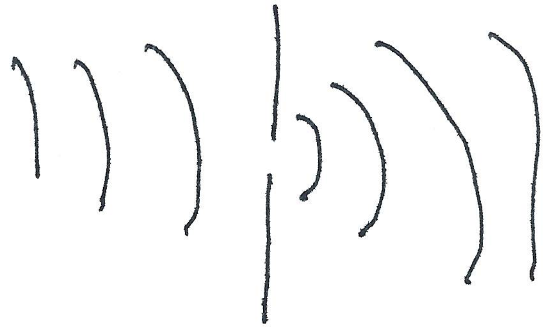
"diffracted field"

$\psi \qquad \hat{n}\cdot\vec{\nabla}\psi$

in the aperture

Can show: if ψ and $\hat{n}\cdot\vec{\nabla}\psi$ both vanish everywhere on any finite surface, and $(\nabla^2 + k^2)\psi = 0$ then $\psi = 0$ identically

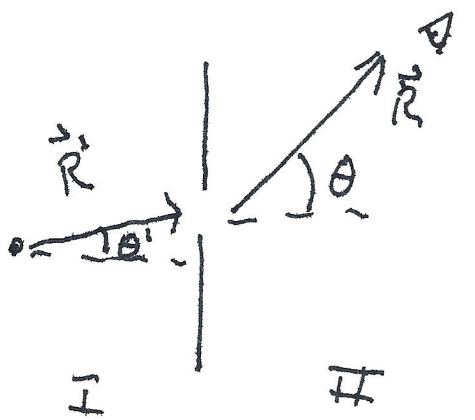
using the wrong Green function



Boundary Conditions match the geometry

Rayleigh - Sommerfeld assumes a planar barrier (except for aperture)

$$G_{\substack{D\\N}} = \frac{1}{4\pi}\left(\frac{e^{ikR}}{R} \mp \frac{e^{ikR'}}{R'}\right)$$



D Dirichlet Boundary

N Neumann Boundary

Turns out

$$\psi(\vec{x}) = \frac{k}{2\pi i}\int \frac{e^{ikR}}{R}\frac{e^{ikR'}}{R'}\,\mathcal{O}(\theta,\theta')\,da$$ R-S

obliquity factor $\mathcal{O}(\theta,\theta') = \begin{cases} \cos\theta & \text{specify } \psi|_{S_1} \\ \cos\theta' & \text{specify } \hat{n}\cdot\vec{\nabla}\psi|_{S_1} \\ \frac{1}{2}(\cos\theta + \cos\theta') & \text{Kirchoff} \end{cases}$

if $|\vec{R}'|, |\vec{R}| \gg d$

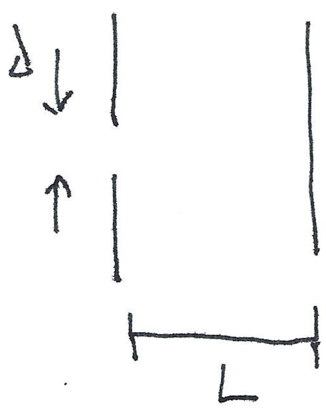
then θ, θ' small

integral by the exponentials, obliquity Factor details are unimportant

Fresnel Diffraction "paraxial diffraction"

"near-field diffraction"

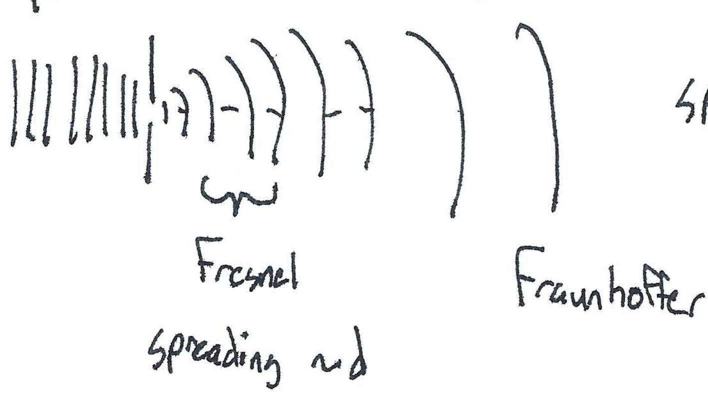
Fresnel number $\sim \frac{Z_R}{L} \sim \frac{d^2}{L\lambda}$



Fresnel Diffraction $F \gtrsim 1$

but also need $F \frac{\theta^2}{4} \ll 1$

"paraxial" near axis



spreading is large Compared to d

$$E(x,y,z) = \frac{1}{i\lambda}\int dx'dy'\, E(x',y',0)\,\frac{e^{ikr}}{R}\,\frac{z}{R}$$

planar barrier at $z=0$ $\qquad k = \frac{2\pi}{\lambda}$

$$R = [\underbrace{(x-x')^2 + (y-y')^2}_{r_\perp^2} + (z-0)^2]^{1/2}$$

$$R = z\sqrt{1 + \frac{r_\perp^2}{z^2}} \simeq z\left(1 + \frac{r_\perp^2}{2z^2} - \frac{1}{8}\left(\frac{r_\perp^2}{z^2}\right)^2 + \dots\right)$$

assume small

$$= z + \frac{r_\perp^2}{2z} + \dots$$

neglect higher order terms

$$k\frac{r_\perp^4}{8z^3} \ll 2\pi$$

implies if $\lambda \ll z$ and $\lambda \ll r_\perp$ then paraxial approximation is good for $r_\perp \ll z$

in e^{ikR} set $R \simeq z + \frac{r_\perp^2}{2z} + \dots$

$$\frac{1}{R} \simeq \frac{1}{z} \qquad \text{good } (x,y) \ll z$$

$$E(x,y,z) \simeq \frac{e^{ikz}}{i\lambda z} \underbrace{\int_{S_1} dx'dy'\, E(x',y',0)}_{\text{modulated}} e^{\frac{ik}{z}\left((x-x')^2+(y-y')^2\right)}$$

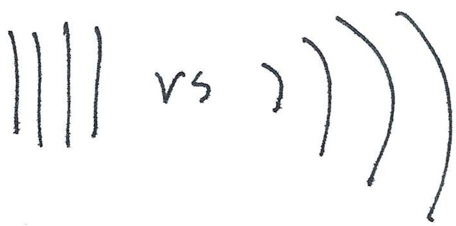
"On-axis" part of a spherical wave propagating out from aperture

accounting for wavefront curvature

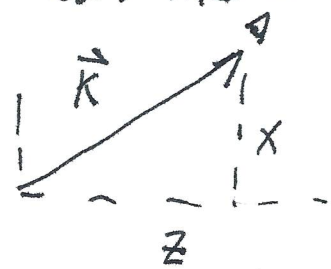
$$= \frac{k e^{ikz}}{iz} e^{i\frac{k}{z}\frac{x^2+y^2}{2}} \frac{1}{2\pi}\int dx'dy'\, E(x',y',0)\, e^{-i(k_x x + k_y y)}$$

angular modulation

where $\cancel{k}$ $k_x = \frac{x}{z}k$ $\quad k_y = k\frac{y}{z}$



looks like a fourier transform



Fraunhofer Diffraction ("Far field diffraction")