Phys 110B 18 Nov 20

Radiation Reaction

non-relativistic case

$$\vec{F}_{rad} \cdot \vec{v} = -P_{larmor} = \frac{-\mu_0 q^2 a^2}{6\pi c}$$

$$\int_{t_1}^{t_2} \vec{F}_{rad} \cdot \vec{v}\, dt = \frac{-\mu_0 q^2}{6\pi c} \left\{ \underbrace{\vec{v} \cdot \frac{d\vec{v}}{dt} \Big|_{t_1}^{t_2}}_{=0} - \int \vec{v} \cdot \frac{d^2\vec{v}}{dt^2} dt \right\}$$

suggests $\vec{F}_{rad} = \frac{\mu_0 q^2}{6\pi c} \dot{\vec{a}}$ Abraham - Lorentz Force

$$\vec{F} = m\vec{a} \rightarrow \vec{F}_{ext} + \vec{F}_{rad} = m\vec{a} = m\ddot{\vec{x}}$$

$$\vec{F}_{ext}(\vec{x}, \vec{v}) \qquad \vec{F}_{rad} \propto \dot{\vec{a}} = \dddot{\vec{x}}$$

changes EOM to a 3rd order differential equation

in particular suppose $\vec{F}_{ext} = \vec{0}$ $\quad \vec{x} = \dot{\vec{x}} = \vec{a} = \vec{0} \quad t < 0$

$$\vec{a}(t) = \begin{cases} \vec{0} \\ \vec{a}_0 e^{t/\tau} \end{cases} \qquad \tau = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{e^2}{mc^3} = \frac{2}{3} \frac{r_e}{c}$$

$q_\bullet$ vs $q_\bullet$ ...

~~pre~~ "runaway acceleration" problem

Can eliminate the "runaway" solutions by imposing

$$\lim_{t \to \infty} \vec{a}(t) = \vec{0} \quad \text{if } \vec{F}_{ext} \text{ turns off}$$

impose via turning ODE into an integral equation

$$m\vec{v}(t) = \int_0^\infty e^{-s} \vec{F}(t + \tau s)\, ds$$

avoids runaways but exhibits "pre-acceleration"

alleleration now depends on acceleration in future

Look at Relativistic case:

$$\frac{d}{d\tau} p^\mu = e F^{\mu\nu} U_\nu = K^\mu_{ext} + K_{rad}$$

$$\frac{d}{d\tau} p^\mu + K^\mu_{ext} + u^\mu \left[\frac{2}{3} \frac{e^2}{c^3 4\pi\epsilon_0} \dot{u}^\alpha \dot{u}_\alpha \right] \qquad \dot{} = \frac{d}{d\tau}$$

Liénard

that "balances" the power loss in time-integrated sense (if we ignore boundary terms)

Inconsistent as a 4-force

$$U^\mu K_\mu \neq 0 \quad \text{in general}$$

Lorentz - Abraham - Dirac Force

only possibility that depends on $u^\mu, \dot{u}^\mu, \ddot{u}^\mu$

$$\dot{p}^\mu = K^\mu_{ext} + K^\mu_{self}$$

$$K^\mu_{self} = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\left[\ddot{u}^\mu - \frac{u^\mu}{c^2}\dot{u}^\alpha\dot{u}_\alpha\right]$$

$$K^\mu_{self} u_\mu = 0$$

$$K^\mu_{self} = K^\mu_{schott} + K^\mu_{rad}$$

$K^\mu_{schott} \downarrow$ $\frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\ddot{u}^\mu$

$K^\mu_{rad} \downarrow$ $-\frac{2}{3}\frac{e^2}{4\pi\epsilon_0}u^\mu\dot{u}^\alpha\dot{u}_\alpha$

total time derivative

reversible

involve near fields

not a total time derivative

irreversible

involves exchange of energy with radiation fields

$$\cancel{F^{\mu\nu} = F^{\mu\nu}_{incident} + \int G\, J^\mu}$$

$$A^\mu = A^\mu_{incident} + \int G_r J^\mu$$

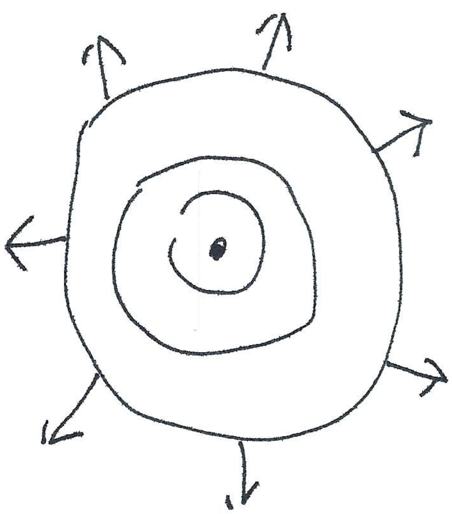
$$A^\mu = A^\mu_{out} + \int G_a J^\mu$$

$$A^\mu_{out} - A^\mu_{in} = \int d^{3+1}x' \left(G_r(\vec{x}-\vec{x}') - G_a(\vec{x}-\vec{x}') \right) J^\mu(\vec{x}')$$

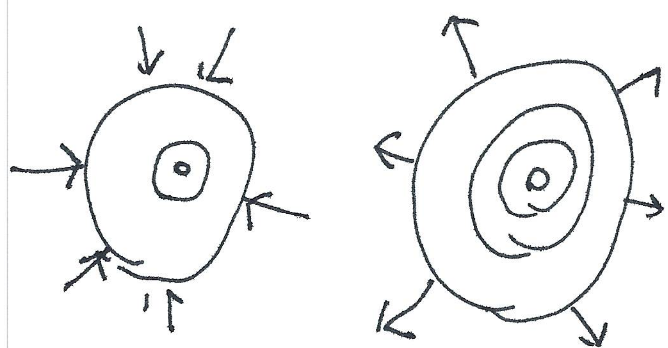
↑ satisfy the source-free Maxwell's equations everywhere

$$A_{out} - A_{in} = A_{rad}$$

"radiation" fields Dirac



outgoing radiation



Dirac Radiation

A_{rad} is regular everywhere (does not blow up at the source)

$$F^{\mu\nu} = F^{\mu\nu}_{ret} = \frac{1}{2}\left(F^{\mu\nu}_{ret} - F^{\mu\nu}_{adv}\right) + \frac{1}{2}\left(F^{\mu\nu}_{ret} + F^{\mu\nu}_{adv}\right)$$

actual fields (near + far) for all sources

$$= \frac{1}{2}F^{\mu\nu}_{rad} + \bar{F}^{\mu\nu}$$

$\frac{1}{2}F^{\mu\nu}_{rad}$: regular everywhere

$\bar{F}^{\mu\nu}$: blows up at point charge sources

$$\dot{p}^{\mu} = K^{\mu}_{ext} + K^{\mu}_{self} - \delta m\, \dot{u}^{\mu}$$

$\dot{p}^{\mu} = m\dot{u}^{\mu}$, $K^{\mu}_{ext} = qF^{\mu\nu}_{ext}u_{\nu}$

δm: infinite constant for a point charge

$$(m + \delta m)\dot{u}^{\mu} = K^{\mu}_{ext} + K^{\mu}_{self}$$

$(m + \delta m)$: from $\bar{F}^{\mu\nu}$

K^{μ}_{ext}: external Lorentz force, from $F^{\mu\nu}_{ext}$

K^{μ}_{self}: Abraham-Lorentz-Dirac self force, from $F^{\mu\nu}_{rad}$

$$(m_{base} + \delta m) = m_{exp} \quad \text{observed mass}$$

m_{base}: without radiation reaction effects

Renormalization of particle mass

$$m_{bare} + \delta m = m_{exp} \text{ is to be finite}$$

$m_{bare} \uparrow \mp\infty$

$\delta m \downarrow \pm\infty$ for a point charge

To avoid pre-acceleration/runaway

Landau-Lifshitz-Spohn

$$m\dot{u}^\mu = q F^{\mu\nu}_{ext} u_\nu + \frac{2}{3}\frac{e^2}{c^3 4\pi\epsilon_0}\left(\frac{q}{m} u^\sigma \partial_\sigma F^{\mu\nu} u_\nu + \left(\frac{q}{m}\right)^2 F^{\mu\alpha} F_\alpha{}^{\nu} u_\nu + \left(\frac{q}{m}\right)^2 \frac{1}{c^2} F^{\sigma\alpha} F_\alpha{}^{\nu} u_\sigma u_\nu u^\mu\right)$$

closest thing we have to a self-consistent ~~form~~ EOM for a classical point charge