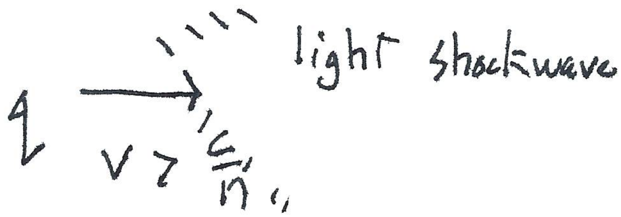
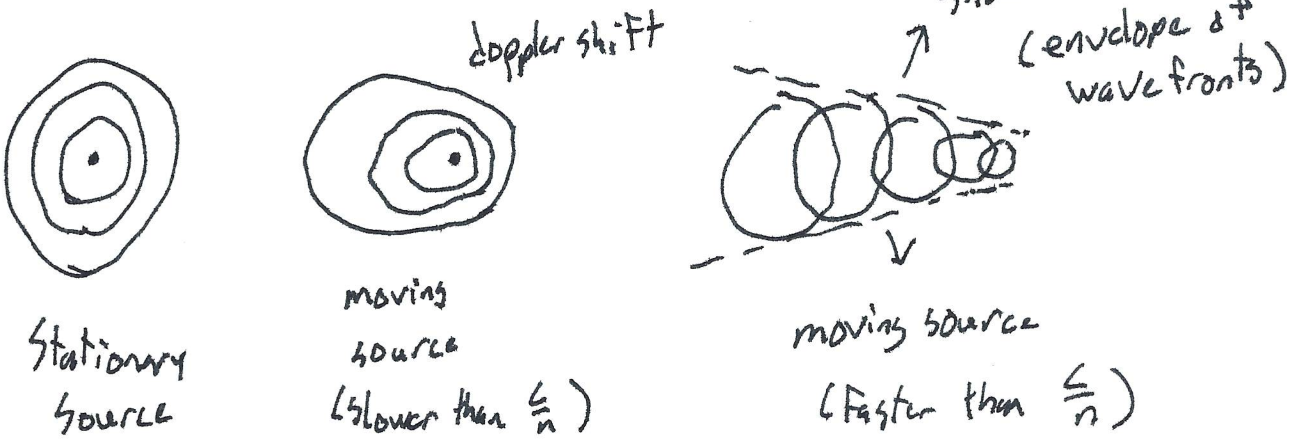
Phys 110B 16Nov20

Midterm Thursday

Cerenkov Radiation ~1934

optical analog to sonic boom





$n = \sqrt{\frac{\epsilon}{\epsilon_0}}$ $\mu \cong \mu_0$

$$\left(\nabla^2 - \frac{n^2}{c^2}\partial_t^2\right)\vec{A} = -\mu_0 \vec{J}$$

looks the same mathematically as the vacuum case except $c \to \frac{c}{n}$

$t_r = t - \frac{Rn}{c}$ retarded time in medium

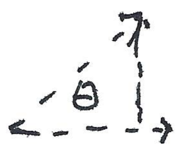
Suppose $\frac{c}{n} < v < c$ when $n > 1$

↑ Source Velocity

$$K = 1 - \frac{vn}{c}\cos\theta \quad \text{can} = 0$$

$\frac{1}{K}$ can blow up → shock wave

$$\Rightarrow \theta = \arccos\left(\frac{c}{vn}\right)$$ well defined real angle

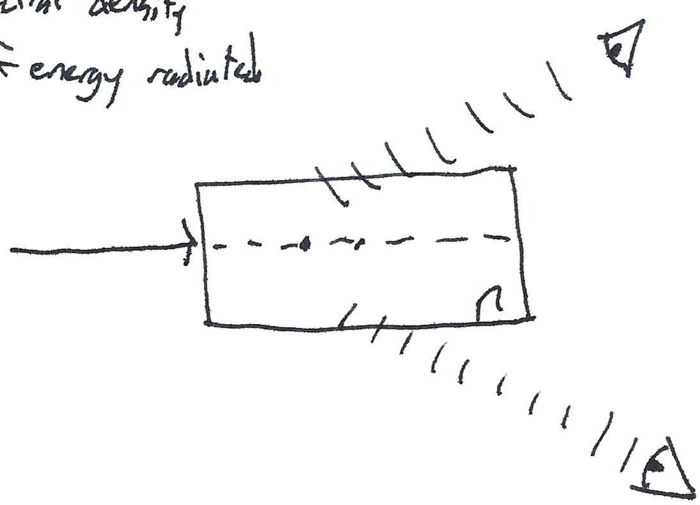


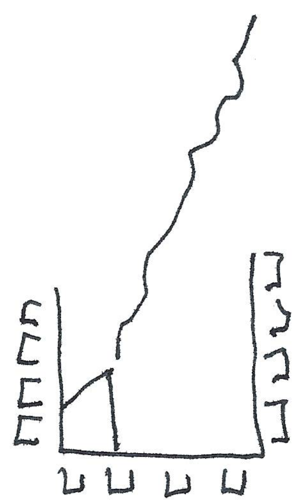
Cerenkov angle

energy radiated at ω

$$\frac{\Delta U_\omega \, d\omega}{\Delta L} = \frac{q^2}{4\pi\epsilon_0 c^2}\left(1 - \frac{c^2}{n^2 v^2}\right)\omega \, d\omega$$

spectral density of energy radiated





Radiation Reaction

given sources $\xrightarrow[\text{Maxwell's Equations}]{}$ $\vec{E}, \vec{B}$ fields

given fields $\vec{E}, \vec{B}$ $\xrightarrow[\text{Lorentz Force Law}]{}$ particle trajectories

} Approximations

Reality is more complicated

Sources tell fields how to point

fields tell sources how to move

self-consistency?

is field/particle division consistent?

EM forces are not reciprocal

conservation laws energy / momentum / angular momentum only hold for the full system

$\sum \vec{P}_i$ in particles $\quad \vec{g} = \dfrac{\vec{E} \times \vec{H}}{c^2}$ momentum in fields

$$u = \frac{\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}}{2}$$

radiation reaction

charged particle accelerates producing radiation

EM radiation carries away energy, momentum, $\vec{L}$, etc.

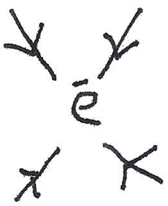
particle must be losing energy, momentum, $\vec{L}$, etc.

(if not being supplied)

Approximations are good when some parameter is small

Small characteristic time-scale

$$\tau_e = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{e^2}{mc^3} = 6 \times 10^{-24}\ s \quad \text{for } e^-$$

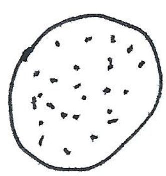


Coulomb fields store energy

Contribute to the rest mass

similarly

Stores electric potential energy



sphere of charge

how big would the charge have to be so that a uniform sphere would store $U_{es} \sim m_e c^2$

$$r_e \sim \frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2}$$

r_e classical electron radius

$\tau \sim$ time required for light in vacuum to traverse ~ 1 classical electron radius

Account for radiation reaction?

low velocity (non-relativistic) case

Larmor $\rightarrow$ $P_{rad} = \frac{\mu_0 q^2 a^2}{6\pi c}$

$\vec{F}_{rad} \cdot \vec{v} = -\frac{\mu_0 q^2 a^2}{6\pi c}$ neglects velocity fields and cross terms

$\int_{t_1}^{t_2} \vec{F}_{rad} \cdot \vec{r}\, dt = -\frac{\mu_0 q^2}{6\pi c} \int_{t_1}^{t_2} \vec{a} \cdot \vec{a}\, dt$ integrate by parts

$= \frac{-\mu_0 q^2}{6\pi c} \left(\vec{v} \cdot \frac{d\vec{v}}{dt} \Big|_{t_1}^{t_2} - \int \vec{v} \cdot \frac{d^2\vec{v}}{dt^2} dt \right)$

if $\vec{v}, \vec{a}, \dot{\vec{a}}$, etc to all vanish at the boundaries

suggests

$\vec{F}_{rad} = \frac{\mu_0 q^2 \dot{\vec{a}}}{6\pi c}$ (Abraham-Lorentz Force)