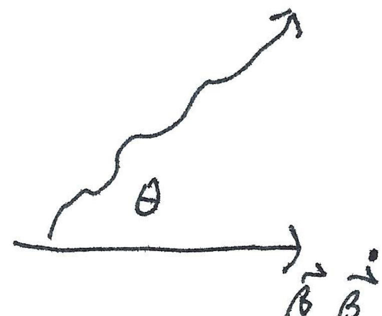
Phys 110B 13 Nov 20

Radiation from Relativistic Sources

$$\frac{dP(t_r)}{d\Omega} = \frac{q^2}{16\pi^2 \epsilon_0 c} \left| \left[\frac{\hat{R} \times ((\hat{R} - \vec{\beta}) \times \dot{\vec{\beta}})}{(1 - \hat{R}\cdot\vec{\beta})^5} \right]_{t_r} \right|^2$$

$$P = \cdots = \frac{\mu_0 q^2 \gamma^6}{6\pi c} \left[|\vec{a}|^2 - |\vec{\beta} \times \vec{a}|^2 \right]_{t_r}$$

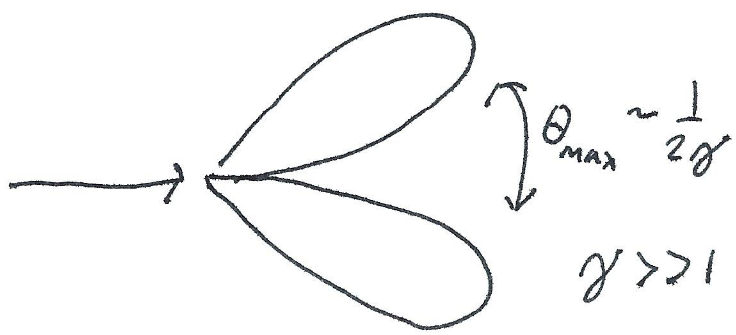
Liénard's Formula



$\vec{\beta}, \dot{\vec{\beta}}$ colinear

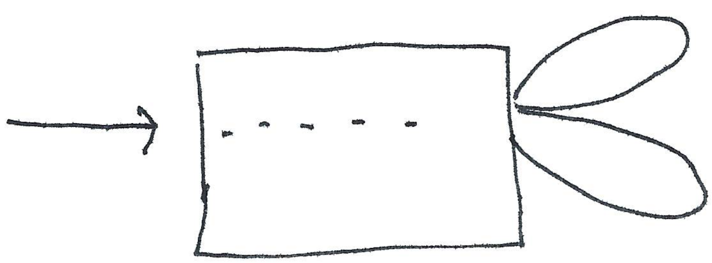
$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2}{16\pi^2 c} a^2 \frac{\sin^2\theta}{(1 - \beta\cos\theta)^5}$$

$$P = \cdots = \frac{\mu_0 q^2 a^2}{6\pi c} \gamma^6$$

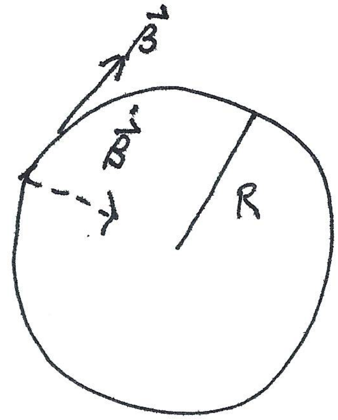


$\langle \theta^2 \rangle^{1/2} \sim \frac{1}{\gamma}$

Bremsstrahlung "breaking radiation"



$\vec{\beta} \perp \dot{\vec{\beta}}$ e.g. synchrotron



E = energy

$$\cancel{\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c^2}\sin^2\theta}$$

$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\left[\frac{(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos\phi}{(1-\beta\cos\theta)^5}\right]_{t_r}$$

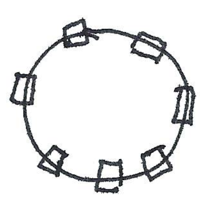
$$P = \cdots = \frac{\mu_0 q^2 a^2}{6\pi c}\gamma^4 = \frac{\mu_0}{6\pi c} q^2 \left(\frac{c^2}{R}\right)^2 \left(\frac{E}{mc^2}\right)^4$$

$\gamma \gg 1$

centripetal acceleration

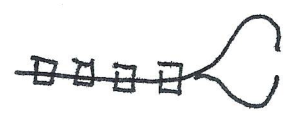
$$P \simeq \frac{\mu_0}{6\pi c} q^2 \frac{1}{m^4 c^2} \frac{E^4}{R^2}$$

CERN



circular accelerator for p^+ p^-

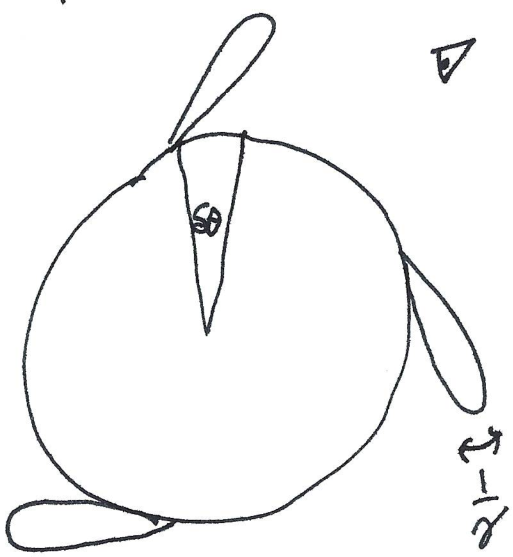
SLAC

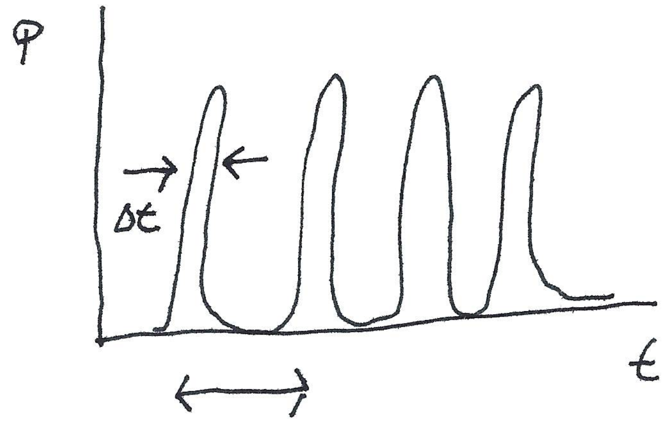


linear accelerator for e^+ and e^-

e^+, e^- too small for circular accelerators. Power loss is too much

Synchrotron Radiation





$$T_0 = \frac{2\pi R}{v} \simeq \frac{2\pi R}{c}$$

$$\delta\theta \sim \frac{1}{\gamma}$$

$$\delta s = R\,\delta\theta$$

$$\Delta t_e = \frac{\Delta s}{v} \Rightarrow c\Delta t_e = \frac{\delta s}{\beta}$$

in time Δt_p Front of pulse ~~~~ moves $c\Delta t_p$

pulse duration $\Delta t_{rad} = \frac{c\Delta t_e - \Delta s}{c} = \frac{\Delta s}{c}\left(\frac{1}{\beta} - 1\right)$

$$\sim \frac{R}{c}\frac{1}{\gamma}\left(\frac{1}{\sqrt{1-\frac{1}{\gamma^2}}} - 1\right)$$

$$\sim \frac{R}{c}\frac{1}{\gamma}\left(1 + \frac{1}{2}\frac{1}{\gamma^2} - 1 + \dots\right)$$

$\delta t_{rad} \sim \frac{R}{c}\frac{1}{\gamma^3}$ (duration of each pulse)

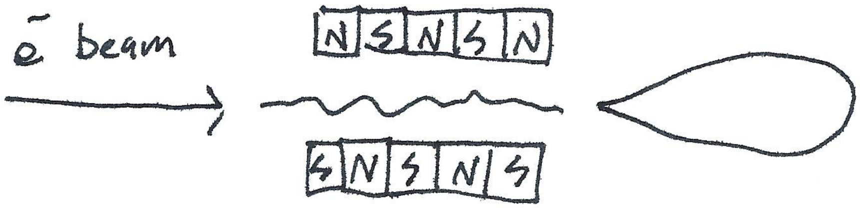
$\Rightarrow$ expect Frequencies $\omega \sim \omega_0 \gamma^3$

Broad spectrum

$$\omega_0 = \frac{2\pi}{T_0}$$

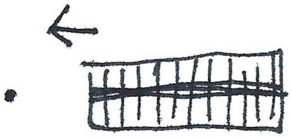
what if we want more collimation narrower spectrum?

Undulator Radiation



$\leftarrow \lambda_u \rightarrow$ undulator period

in average rest frame of the beam



Lorentz contraction of magnets

$\rightsquigarrow$ dipole radiatio

back in lab frame

doppler shift + headlighting

Resonance argument

$\lambda_u = v_e \Delta t$ In Δt, radiation $c\Delta t = \lambda_u + \lambda$

resonance/
constructive
interference

$$\frac{\lambda_u}{v} = \frac{\lambda}{c} + \frac{\lambda_u}{c} \Rightarrow \frac{\lambda}{c} = \frac{\lambda_u}{c}\left(\frac{1}{\beta} - 1\right) \Rightarrow \lambda = \lambda_u \frac{(1-\beta^2)}{\beta(1+\beta)} \approx \frac{\lambda_u}{2\gamma^2}$$

upshift in frequency

Relativistic beams $\gamma \sim 10^3 - 10^6$

$\lambda_u \sim$ cm $\qquad \lambda \sim$ x-ray

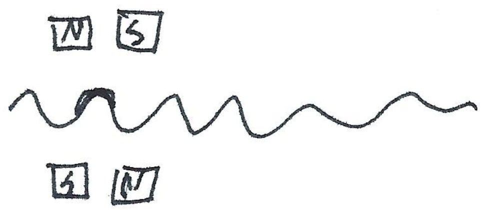
bandwidth

N wiggles

$$\frac{\Delta\omega}{\omega} = \frac{1}{N}$$

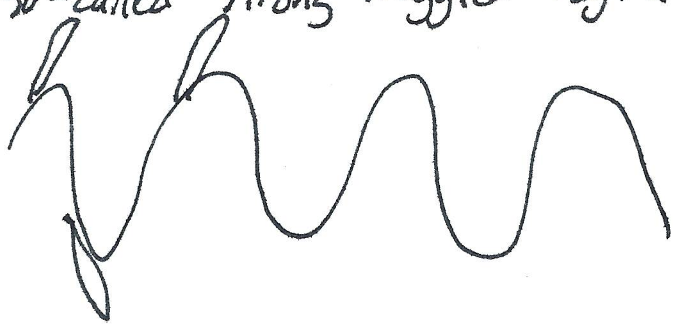
relative bandwidth

angular spread?



$\delta\theta \sim \frac{1}{\gamma}$?

true in so-called strong wiggler regime



like synchrotron radiation

weak undulator regime

more gentle oscillations

angular deviations of e^- small compared to $\frac{1}{\gamma}$

Overlapping "cones" of radiation in electron beam

$\Rightarrow$ destructive interference

$\Rightarrow \Delta\theta \sim \dfrac{1}{\sqrt{N}\,\gamma}$ for "coherent mode" of radiation

Cerenkov Radiation ~ 1934

"optic Boom"

"shock wave" of light