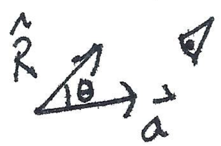
Phys 110B 09Nov20

$$\vec{E}_{rad} = \frac{q}{4\pi\epsilon_0}\left[\frac{\hat{R}\times(\hat{R}-\vec{\beta})\times\dot{\vec{\beta}}}{(1-\vec{\beta}\cdot\hat{R})^3 cR}\right]_{ret}$$

$$\vec{B}_{rad} = \left[\frac{\hat{R}\times\vec{E}}{c}\right]_{ret}$$

$$\vec{S}_{rad} = \frac{1}{\mu_0 c}\left|\vec{E}_{rad}\right|^2\hat{R}$$

Particle is instantaneously at rest



$\vec{\beta} = \vec{0}$ instantaneously

$$R^2\vec{S}_{rad}\cdot\hat{R} = \frac{dP}{d\Omega} = \cdots = \frac{\mu_0 q^2}{16\pi^2 c}a^2\sin^2\theta$$

$$P = \int\frac{dP}{d\Omega}d\Omega = \frac{\mu_0 q^2}{6\pi c}a^2 \quad \text{Larmor Formula}$$

$d\Omega = \sin\theta\, d\theta\, d\phi$ $\qquad \frac{1}{4\pi\epsilon_0}\frac{2}{3}q^2a^2$

Alternate derivation

recall for an electric dipole $\omega^2\vec{p} \propto \ddot{\vec{p}}$

far-field

$$\vec{E} = \frac{\mu_0}{4\pi}\frac{1}{r}\left[\hat{r}\times(\hat{r}\times[\ddot{\vec{p}}])\right] \propto \frac{\mu_0}{4\pi}[\ddot{p}]\frac{\sin\theta}{r}\hat{\theta}$$

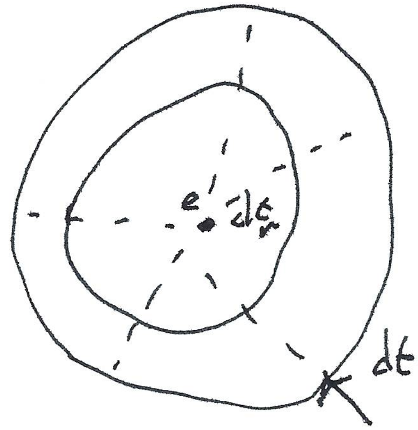
$$\vec{B} = \frac{-\mu_0}{4\pi c}\frac{1}{r}\left[\hat{r}\times[\ddot{\vec{p}}]\right] \propto \frac{\mu_0}{4\pi}[\ddot{p}]\frac{\sin\theta}{r}\hat{\phi}$$

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{\mu_0}{16\pi^2 c}[\ddot{p}]\,[\ddot{p}]\,\frac{\sin^2\theta}{r^2}\,\hat{r}$$

$$\Rightarrow P = \frac{\mu_0}{6\pi c}[\ddot{p}]$$

$p \sim qd$

$\ddot{p} \sim q\ddot{d} \sim qa$



$$P = \frac{\mu_0 q^2}{6\pi c} a^2 \qquad \text{Larmor Formula}$$

(Relativistic) Radiated Power

$$\frac{dP}{d\Omega} = \underbrace{R^2 \vec{S}\cdot\hat{R}}_{\text{Power observed per solid angle}} \cdot \frac{dt}{dt_r} = R^2\vec{S}\cdot\hat{R}\,[1-\vec{\beta}\cdot\hat{R}]$$

if $\vec{\beta} = \vec{0}$ then $dt_r = dt$

if $\vec{\beta} \neq \vec{0}$ then $K \neq 0$

Energy per observer time per solid angle

Not a natural measure of particle's radiation

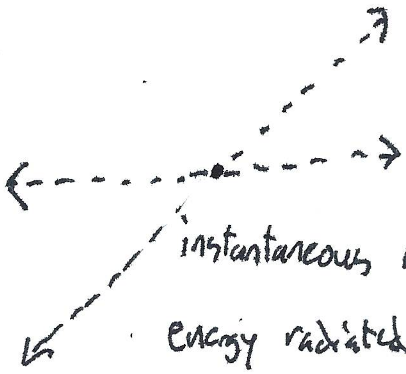
$$\frac{dP(t_r)}{d\Omega} = \cdots = \frac{q^2}{16\pi^2\epsilon_0 c}\left\| \left[\frac{\hat{R}\times((\hat{R}-\vec{\beta})\times\dot{\vec{\beta}})}{(1-\hat{R}\cdot\vec{\beta})^5}\right]_{t_r}\right\|^2$$

$$P(t_r) = \cdots = \frac{\mu_0 q^2}{6\pi c}[\gamma^6]\left[|\vec{a}|^2 - |\vec{\beta}\times\vec{a}|^2\right] \geq 0$$

relativistic generalization of Larmor Formula

Liénard's Formula

* Emitted power (radiated by a point charge) must be a Lorentz Invariant



instantaneous rest Frame (primed)

· energy radiated is the same in antiparallel directions

$$\begin{bmatrix} \frac{dE'}{c} \\ dp'_{\parallel} \\ dp'_{\perp} \end{bmatrix}$$

← energy radiated in a time dt'

← 4-vector

$$\begin{bmatrix} \gamma & \beta\gamma & 0 \\ \beta\gamma & \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dE'/c \\ dp'_{\parallel} \\ dp'_{\perp} \end{bmatrix} = \begin{bmatrix} dE/c \\ dp_{\parallel} \\ dp_{\perp} \end{bmatrix}$$

↑ back to lab frame

$$dE = \gamma\left(dE' + \beta c\, dp'_{\parallel}\right)$$

↑ this term goes away as it averages to zero

$$dE = \gamma\, dE'$$

lab Frame

proper frame of source

$$dt' = d\tau = \frac{dt}{\gamma} \qquad dt = \gamma\, dt'$$

$$\cancel{\frac{dE'}{dt'}} = \frac{dE}{dt} = \frac{\gamma\, dE'}{\gamma\, dt'} = \frac{dE'}{dt'}$$

$\therefore \frac{dE}{dt}$ is a Lorentz Scalar

emitted power in lab frame

emitted power in rest Frame

Quantum Mechanically, the number of emitted photons per proper time interval must be invariant.

1) Emitted power is a Lorentz scalar

2) $e, m, q, p^\mu, \frac{d}{d\tau}p^\mu$, etc.
of source

3) reduce to Larmor Formula as $|\vec{p}| \to 0$

the only possibility

$$P = \xi \frac{dp^\mu}{d\tau}\frac{dp_\mu}{d\tau}$$

Scalar depending on $c, m, q, \ldots$

agreement with Larmor $\rightarrow$ $\xi = \frac{\mu_0 q^2}{6\pi m^2 c}$

Agrees with earlier expression for Liénard Formula

radiated energy-momentum Δp^μ must be a 4-vector

$$\frac{d}{d\tau}\Delta p^\mu = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2}{c^5}\, a^\nu a_\nu U^\mu$$

emitter's proper time

4-acceleration of particle

4-velocity of source

$$P = \frac{\mu_0 \varepsilon^2}{6\pi c}\left[\gamma^6\right]\left[|\vec{a}|^2 - |\vec{\beta} \times \vec{a}|^2\right]$$

Case 1: Bremsstrahlung - "braking radiation"

$\vec{\beta}, \dot{\vec{\beta}}$ colinear (parallel or antiparallel)