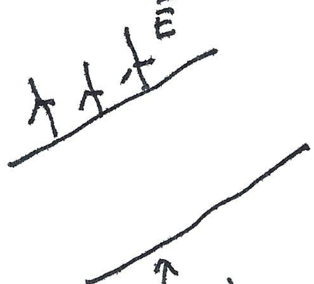
Phys 110B 06Nov20

"Antenna Paradox"

use a good conductor for efficiency

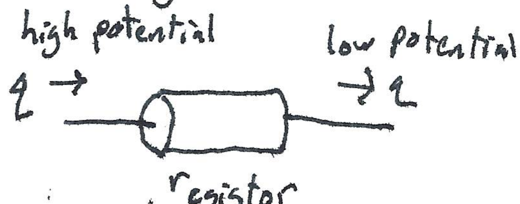


↑ good conductor

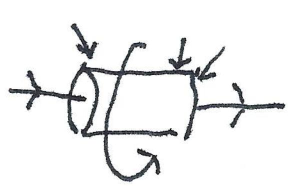
$$\hat{n}\cdot\vec{S}\Big|_{\text{surface of conductor}} \propto (\vec{E}\times\vec{B})\cdot\hat{n} \propto (\hat{n}\times\vec{E})\cdot\vec{B} = 0$$

No Poynting flux out of the antenna!

Antenna is more like a guide of radiation



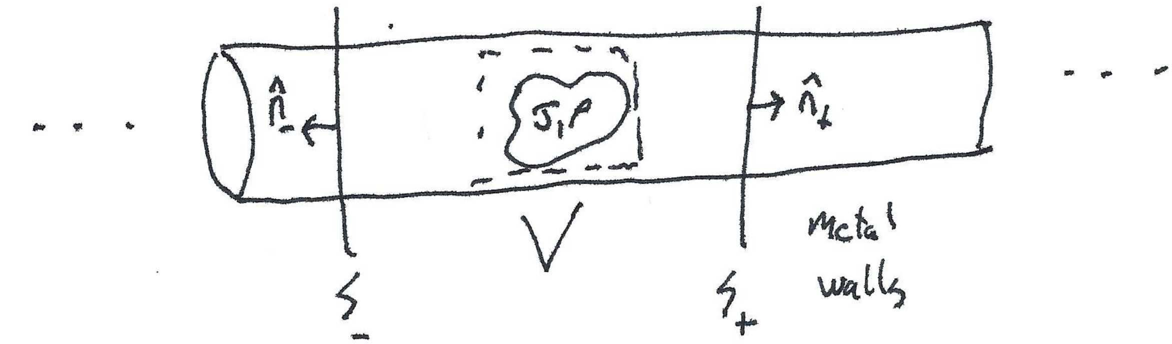
fields



$\vec{S}$ is radial in towards the resistor

Possibly taking our book-keeping too seriously

Radiation by a localized source into a waveguide



Localized $\vec{J} \sim e^{-i\omega t}$ prescribed source

+ right moving waves
− left moving waves

$$\vec{E}_j^{+}(\vec{x};\omega) = \Big(\underbrace{\vec{E}_j(x,y;\omega)}_{\text{real transverse profile}} + \cancel{\hat{z}E_{jz}(x,y;\omega)}\Big)\, e^{ik_j z} \longrightarrow i\hat{z}\,E_{jz}(x,y;\omega)$$

$\omega = \omega_j(k_j)$

$$\vec{H}_j^{+}(\vec{x};\omega) = \left(\vec{H}_j + \hat{z}\,H_{jz}\right) e^{ik_j z}$$

$$\vec{E}_j^{-}(\vec{x};\omega) = \left(\vec{E}_j - \hat{z}\,E_{jz}\right) e^{-ik_j z}$$

$$\vec{H}_j^{-}(\vec{x};\omega) = \left(-\vec{H}_j + \hat{z}\,H_{jz}\right) e^{-ik_j z}$$

Can also choose $\frac{1}{2}\int da\, \hat{z}\cdot(\vec{E}_j \times \vec{H}_\ell) = \delta_{j\ell}\, \frac{1}{2Z_j}$ ← wave impedance

"reciprocity" relationships

$$\vec{\nabla}\cdot\Big(\underbrace{\vec{E}}_{\substack{\text{actual fields}\\ \text{in presence of the sources}}} \times \underbrace{\vec{H}_j^{\pm}}_{\text{wave guide modes}} - \vec{E}_j^{\pm} \times \vec{H}\Big)$$

$$\vec{\nabla}\cdot\left(\vec{E}\times\vec{H}_j^{\pm}-\vec{E}_j^{\pm}\times\vec{H}\right)$$

$$=\underbrace{\vec{H}_j^{\pm}\cdot(\vec{\nabla}\times\vec{E})}_{=\,i\omega\vec{B}\,=\,\mu\vec{H}}-\underbrace{\vec{E}\cdot(\vec{\nabla}\times\vec{H}_j^{\pm})}_{\vec{0}+\epsilon_0 i\omega\vec{E}_j^{\pm}}-\underbrace{\vec{H}\cdot(\vec{\nabla}\times\vec{E}_j^{\pm})}_{=\,i\omega B_j^{\pm}}+\underbrace{\vec{E}_j^{\pm}\cdot(\vec{\nabla}\times\vec{H})}_{\vec{J}+\epsilon_0 i\omega\vec{E}}$$

$$=\cdots=\vec{J}\cdot\vec{E}_j^{\pm}$$ Lorentz reciprocity formula

integrate over region V

$$\int_{S_-+S_++S_w}\left(\vec{E}\times\vec{H}_j^{\pm}-\vec{E}_j^{\pm}\times\vec{H}\right)\cdot\hat{n}\,da=\int_V d^3x\,\vec{J}\cdot\vec{E}_j^{\pm}$$

$\vec{E}_j^{\pm}\big|_{S_w}\propto\hat{n}\big|_{S_w}$ no contribution on left from the walls S_w

$\vec{E}\big|_{S_w}\propto\hat{n}\big|_{S_w}$

on S_+ $\quad \vec{E}_\perp=\sum_j a_j^+\vec{E}_j^+e^{ik_jz} \quad \hat{n}\big|_{S_+}\propto\hat{z}$

(a_j^+: expansion coefficients)

on S_- $\quad \vec{E}_\perp=\sum a_j^-\vec{E}_j^-e^{-ik_jz} \quad \hat{n}\big|_{S_-}\propto-\hat{z}$

$$\int_V\vec{J}\cdot\vec{E}_l^{+}d^3x'=\sum_j a_j^+\int_{S_+}\left(\vec{E}_j^+\times\vec{H}_l^- -\vec{E}_l^-\times\vec{H}_j^+\right)\cdot\hat{z}\,da$$

$$-\sum_j a_j^-\int_{S_-}\left(\vec{E}_j^-\times\vec{H}_l^- -\vec{E}_l^-\times\vec{H}_j^-\right)\cdot\hat{z}\,da\to 0$$ due to orthogonality

$$\int\vec{J}\cdot\vec{E}_l^-d^3x'=\sum_j a_j^+\int_{S_+}\left(\vec{E}_j^+\times\vec{H}_l^- -\vec{E}_l^-\times\vec{H}_j^+\right)\cdot\hat{z}\,da=*$$

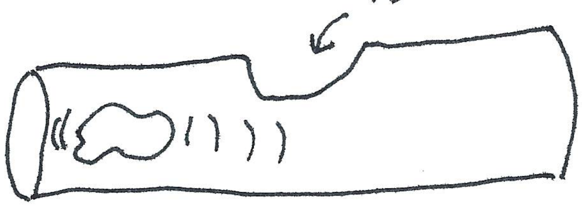
$$\Rightarrow a_\ell^{\pm} = -\frac{Z_\ell}{2}\int_V \vec{J}\cdot \vec{E}_\ell^{\mp}\, d^3\vec{x}$$

"Joule" work integral

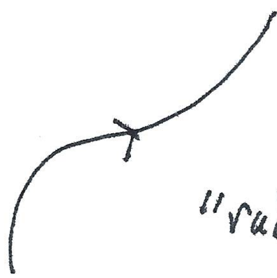
overlap integral
over the source

current density $\vec{J}$ tends to radiate into the modes that "look" most like $\vec{J}$

hole



Radiation from relativistic sources (mostly into vacuum)



"radiation" fields

$$\vec{E}_{rad} = \frac{q}{4\pi\epsilon_0 c^2}\left[\frac{\vec{R}\times(\vec{R}-R\vec{\beta})\times\dot{\vec{u}}}{(R-\vec{\beta}\cdot\vec{R})^3}\right]$$

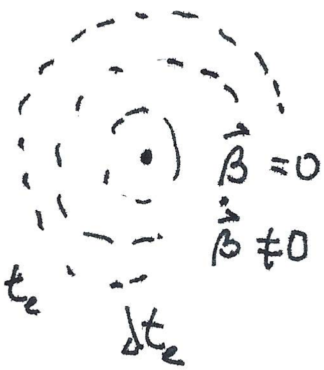
$$\vec{B}_{rad} = \left[\frac{\hat{R}\times\vec{E}}{c}\right]$$

$$\Rightarrow \vec{E}_{rad} = \frac{q}{4\pi\epsilon_0}\left[\frac{\hat{R}\times(\hat{R}-\vec{\beta})\times\dot{\vec{\beta}}}{(1-\vec{\beta}\cdot\hat{R})^3 cR}\right]_{ret}$$

$$\vec{B}_{rad} = \left[\frac{\hat{R}\times\vec{E}}{c}\right]$$

$$\vec{S}_{rad} = \frac{1}{\mu_0 c}\left|\vec{E}_{rad}\right|^2 \hat{R}$$ far-field

For a charge instantaneously at rest



look over some large sphere at some suitable later time

$$\frac{dP}{d\Omega} = R^2 \vec{S}_{rad} \cdot \hat{R} = \frac{\mu_0 q^2 a^2 \sin^2\theta}{16\pi^2 c}$$

$\vec{a} = \dot{\vec{v}}$ = acceleration of charge