Phys 110B 26 Oct 20

Liénard - Wiechart Fields (from moving point charge)

$$\phi = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R(1-\hat{R}\cdot\vec{\beta})}\right]_{ret} \qquad \vec{R} = \vec{x} - \vec{r}(t)$$

$$\vec{A} = \frac{\mu_0}{4\pi} q \left[\frac{\vec{\beta}}{cR(1-\hat{R}\cdot\vec{\beta})}\right]_{ret}$$

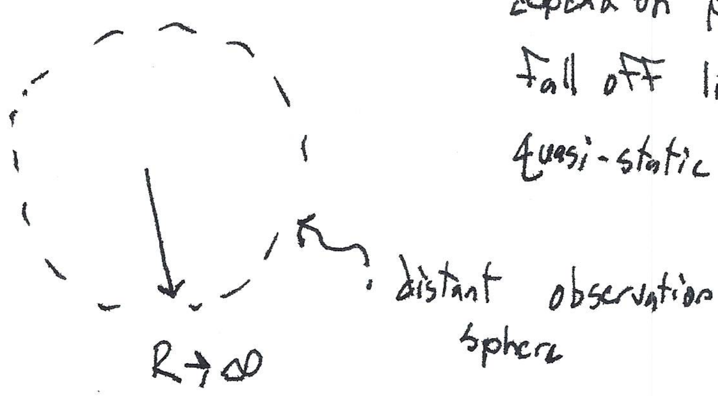
acceleration fields
depend on $\vec{\beta}, \dot{\vec{\beta}}$
fall off like $\frac{1}{R}$

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{(1-\beta^2)(\hat{R}-\vec{\beta})}{R^2(1-\hat{R}\cdot\vec{\beta})^3}\right]_{ret} + \frac{q}{4\pi\epsilon_0}\left[\frac{\hat{R}\times\left[(\hat{R}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{cR(1-\hat{R}\cdot\vec{\beta})}\right]_{ret}$$

radiative or radiation fields

$$\vec{B} = \left[\frac{\hat{n}\times\vec{E}}{c}\right]_{ret} \qquad t_r = t - \frac{R}{c} \text{ implicit}$$

velocity fields
depend on $\vec{\beta}$ not $\dot{\vec{\beta}}$
fall off like $\frac{1}{R^2}$
quasi-static



Radiation is associated with the acceleration of charge

Away from the worldline of the source charge.

Satisfy the source-free Maxwell's Equations

$\vec{E} \cdot \vec{B} = 0$ Lorentz invariant

$\left[\hat{R} \cdot \vec{B}\right]_{ret} = 0$

True for full field off the worldline

$\left[\hat{R}\right]_{ret} \cdot \vec{B} = 0$

$\left[\hat{R}\right]_{ret} \times \vec{E} \propto \vec{B}$

in "far-zone" (far away from the source)

acceleration fields dominate the velocity fields

$|\vec{E}|^2 - c^2|\vec{B}| = 0$

$\left[\hat{R}\right]_{ret} \cdot \vec{E} = 0$

In the far-zone, all 6 properties hold

$\left[\hat{R}\right]_{ret} \times \vec{B} \propto -\vec{E}$

$\kappa = \left[1 - \vec{\beta} \cdot \hat{R}\right]_{ret}$

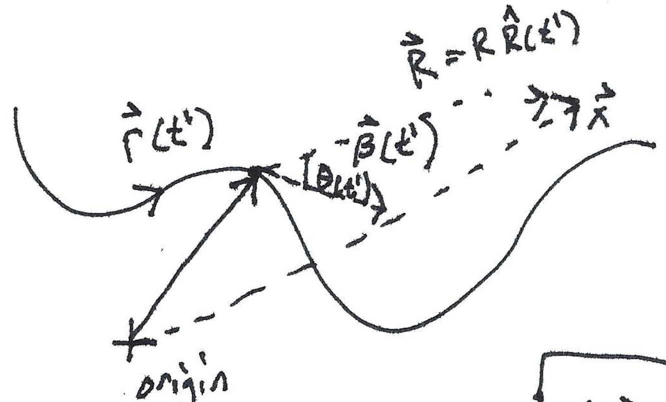
particle in uniform motion $\int \rho(\vec{x}', t) d^3\vec{x}' = q$

then $\int \rho(\vec{x}', t_r) d^3\vec{x}' = \frac{q}{1 - \vec{\beta} \cdot \hat{R}(t)}$

spatial distortion of apparent charge distribution

κ = temporal compression factor

scale factor between changes in observation time dt and the emitter time dt'



$$t = t' + \sqrt{\frac{|\vec{x} - \vec{r}(t')|^2}{c^2}}$$

$$k = \frac{\Delta t}{\Delta t'} = 1 + \frac{1}{c}\,\frac{\frac{1}{2}\,2\,(-1)\,\frac{\Delta \vec{r}}{\Delta t'}\,(\vec{x} - \vec{r}(t'))}{|\vec{x} - \vec{r}(t')|}$$

$$= 1 - \vec{\beta}(t') \cdot \hat{R}(t') = 1 - \beta(t')\cos\theta(t')$$

Heart of doppler effects and headlight effects

Highly relativistic limit
↑
source

$$1 - \beta = 1 - \sqrt{1 - \frac{1}{\gamma^2}} \approx \frac{1}{2\gamma^2} \ll 1$$

$$k \cong 1 - \left[1 - \frac{1}{2\gamma^2}\right]_{ret}\left[\cos\theta\right]_{ret} \approx 1 - [\cos\theta]_{ret} + \left[\frac{1}{\gamma^2}\cos\theta\right]_{ret} + \ldots$$

when θ is small, $\cos\theta \approx 1 - \frac{1}{2}\theta^2 + \ldots$

$$k = \left[\frac{1}{2\gamma^2}\right]_{ret} + \left[\frac{1}{2}\theta^2\right]_{ret} + \ldots \ll 1$$ A lot of compression

when $\theta \leq \frac{\pi}{2}$ $\quad$ θ near $\frac{\pi}{2}$

$k \approx 1$ $\quad$ ~~A lot of compression~~ Not much compression

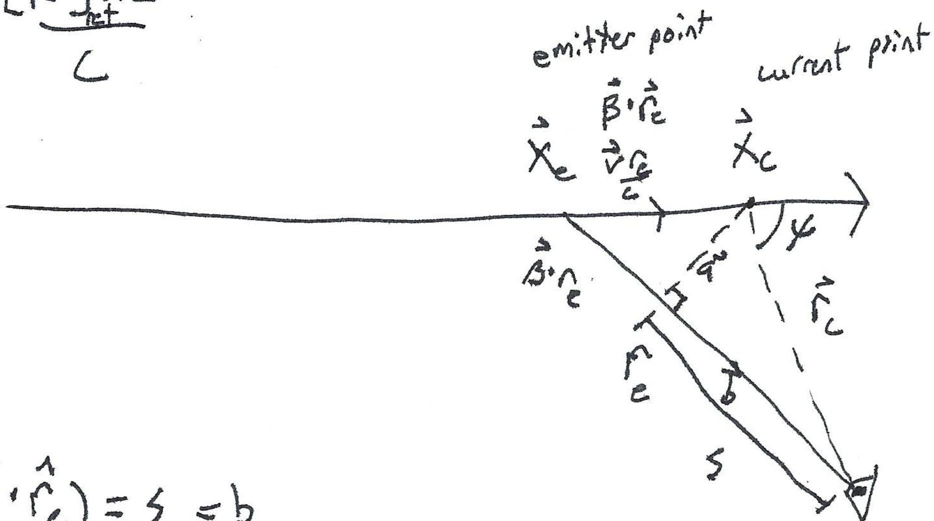
Fields from uniformly moving charge

$\vec{\beta} = \text{constant} \qquad \dot{\vec{\beta}} = 0$

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{(1-\beta^2)(\hat{R}-\vec{\beta})}{R^2(1-\hat{R}\cdot\vec{\beta})^3}\right]_{ret}$$

only velocity fields

$$\vec{B} = \frac{[\hat{R}]_{ret}\times\vec{E}}{c}$$



$r_e(1-\vec{\beta}\cdot\hat{r}_e) = s = b$

$\vec{r}_e = \vec{x} - \vec{x}_e$

$\vec{r}_c = \vec{x} - \vec{x}_c$

$\vec{\beta} r_e + \vec{r}_c = \vec{r}_e$

$a = |\hat{r}_e \times \vec{\beta} r_e| = |\vec{r}_e \times \vec{\beta}|$

$= |\vec{r}_c \times \vec{\beta}|$

$a^2 + b^2 = r_c^2$

$b^2 = r_c^2[1-\beta^2\sin^2\psi]$

$$\vec{E}(\vec{x},t) = \frac{q}{4\pi\epsilon_0}\left[\hat{r} - \beta\hat{r}\right]_{ret}(1-\beta^2) = \frac{q}{4\pi\epsilon_0}\frac{\vec{r}_c(1-\beta^2)}{b^3}$$

$$= \frac{q}{4\pi\epsilon_0}\frac{\vec{r}_c}{r_c^3}\frac{(1-\beta^2)}{(1-\beta^2\sin^2\psi)^{3/2}}$$

$$\vec{B} = \frac{1}{c}\vec{\beta}\times\vec{E}$$