Phys 110B 23Oct20

Jefimenko Expressions

$$\vec{E}(\vec{x},t) = \frac{1}{4\pi\epsilon_0}\int d^3\vec{x}'\left(\frac{\hat{R}}{R^2}\left[\rho(\vec{x}',t')\right]_{ret} + \frac{\hat{R}}{cR}\left[\frac{\partial}{\partial t'}\rho(\vec{x}',t')\right]_{ret} - \frac{1}{c^2}\frac{1}{R}\left[\frac{\partial}{\partial t}\vec{J}(\vec{x}',t')\right]_{ret}\right)$$

$$\vec{R} = \vec{x} - \vec{x}' \qquad R = |\vec{R}| \qquad \hat{R} = \frac{\vec{R}}{R}$$

$$\vec{B}(\vec{x},t) = \frac{\mu_0}{4\pi}\int d^3\vec{x}'\left(\left[\vec{J}(\vec{x}',t')\right]_{ret} \times \frac{\hat{R}}{R^2} + \left[\frac{\partial}{\partial t}\vec{J}(\vec{x}',t')\right]_{ret} \times \frac{\hat{R}}{cR}\right)$$

Panofsky - Phillips K. MacDonald

$$\vec{E}(\vec{x},t) = \frac{1}{4\pi\epsilon_0}\int d^3\vec{x}'\,\frac{[\rho]_{ret}\hat{R}}{R^2}$$

$$+ \frac{1}{4\pi\epsilon_0}\int d^3\vec{x}'\,\frac{([\vec{J}]_{ret}\cdot\hat{R})\hat{R} + ([\vec{J}]_{ret}\times\hat{R})\times\hat{R}}{cR^2}$$

← currents produce electric fields?

vanishes in steady state

$$+ \frac{1}{4\pi\epsilon_0}\int d^3\vec{x}'\,\frac{\left(\left[\frac{\partial}{\partial t'}\vec{J}\right]_{ret}\times\hat{R}\right)\times\hat{R}}{c^2 R}$$

$$\frac{\partial}{\partial t}\vec{J}(\vec{x},t) = \vec{0}$$

$$\vec{\nabla}\cdot\vec{J}(\vec{x},t) = 0$$

Fields from point charges

$$\rho(\vec{x},t) = q\,\delta(\vec{x}-\vec{r}(t)) \qquad \vec{J}(\vec{x},t) = \vec{v}(t)\rho(\vec{x},t)$$

with trajectory $\vec{r}(t)$

$$\Rightarrow \vec{E} = \frac{q}{4\pi\epsilon_0}\left(\left[\frac{\hat{R}}{\kappa R^2}\right]_{ret} + \frac{\partial}{c\partial t}\left[\frac{\hat{R}}{\kappa R}\right]_{ret} - \frac{\partial}{c^2\partial t}\left[\frac{\vec{v}}{\kappa R}\right]_{ret} \right)$$

$$\vec{B} = \frac{\mu_0 q}{4\pi}\left(\left[\frac{\vec{v}\times\hat{R}}{\kappa R^2}\right]_{ret} + \frac{\partial}{c\partial t}\left[\frac{\vec{v}\times\hat{R}}{\kappa R}\right]_{ret} \right)$$

$\vec{v} = \dot{\vec{r}}$ velocity of charge

$|\vec{R}| = |\vec{x} - \vec{r}(t_r)| \qquad t_r = t - \frac{|\vec{x}-\vec{r}(t_r)|}{c}$

κ = "time compression factor" $= 1 - \frac{\vec{v}\cdot\hat{R}}{c} = 1 - \vec{\beta}\cdot\hat{R}$

$\Rightarrow \cdots \Rightarrow$ Feynman-Heaviside Expressions for fields of a moving point charge

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\left(\left[\frac{\hat{R}}{R^2}\right]_{ret} + \frac{[R]_{ret}}{c}\frac{\partial}{\partial t}\left[\frac{\hat{R}}{R^2}\right]_{ret} + \frac{\partial^2}{c^2\partial t^2}\left[\hat{R}\right]_{ret} \right)$$

$$\vec{B} = \frac{\mu_0 q}{4\pi}\left(\left[\frac{\vec{v}\times\hat{R}}{\kappa^2 R^2}\right]_{ret} + \frac{1}{c[R]_{ret}}\frac{\partial}{\partial t}\left[\frac{\vec{v}\times\hat{R}}{\kappa}\right]_{ret} \right)$$

$\underline{\vec{E}}$ First term is Coulomb's Law evaluated at retarded time

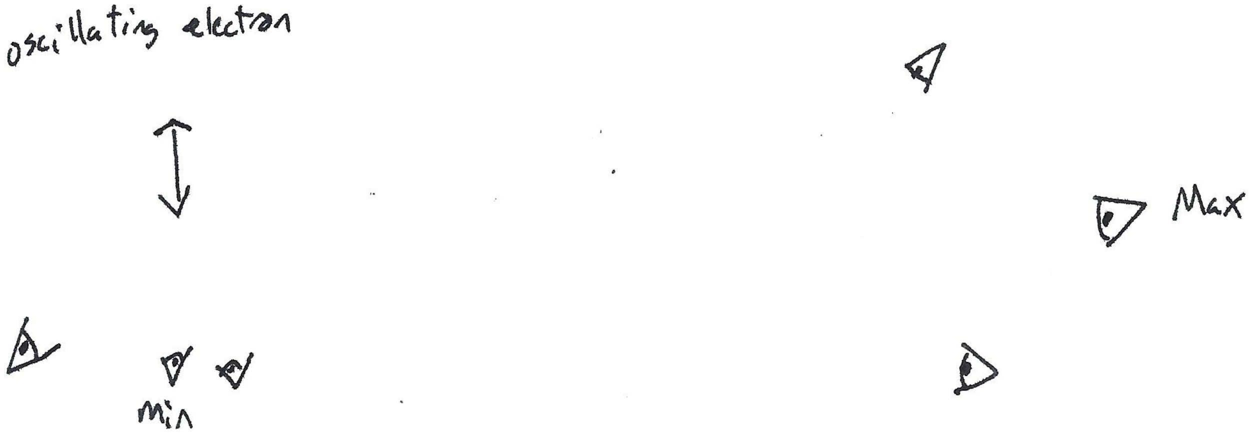
Second term is like a 1st order term in Taylor series correction for motion

Third term

$$\frac{q}{4\pi\epsilon_0}\frac{\partial^2}{c^2\partial t^2}[\hat{R}]_{ret} = \text{Radiation}$$

radiation is associated with the acceleration of charges

Radiation is nature's way of communicating changes in the state of motion of sources to remote locations.



Different angles of observer see a projection

Magnetic radiation term

$$\vec{B} = \frac{[\hat{R} \times \vec{E}]_{ret}}{c}$$

radiation must locally look like a plane wave

Liénard - Wiechert Potentials

Lorentz-Lorenz gauge potentials due to a point charge

$$\phi = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{RK}\right]_{ret}$$

$K = 1 - \hat{n}\cdot\vec{\beta}$ ← of the particle

$$\vec{A} = \frac{\mu_0 q}{4\pi}\left[\frac{\vec{\beta}}{cRK}\right]_{ret}$$

*1 *2

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial}{\partial t}\vec{A} = \ldots = \frac{q}{4\pi\epsilon_0}\left[\frac{(1-\beta^2)(\hat{R}-\vec{\beta})}{R^2(1-\hat{n}\cdot\vec{\beta})^3}\right]_{ret} + \frac{q}{4\pi\epsilon_0}\left[\frac{\hat{R}\times((\hat{R}-\vec{\beta})\times\dot{\vec{\beta}}}{cR(1-\hat{n}\cdot\vec{\beta})^3}\right]_{ret}$$

$$\cancel{\vec{B} = \ldots = [\hat{R}\times\vec{E}]} \quad \cancel{[\hat{R}]\times\vec{E}}$$

$$\vec{B} = \ldots = \left[\frac{\hat{R}\times\vec{E}}{c}\right]_{ret}$$

*1 falls off like $\frac{1}{R^2}$ depends on $\vec{\beta}$ but not $\dot{\vec{\beta}}$ "velocity fields" non-radiative

*2 falls off like $\frac{1}{R}$ depends on $\dot{\vec{\beta}}$ "acceleration fields" radiative