Phys 110B 14 Oct 20

Midterm Thursday 10 am to
Friday 10 am
3 hour window
Bcourses quiz

"open-bcourses"
No other
resources

Covers Special Relativity
Relativistic Electrodynamics
Covariant Electrodynamics
Bases of waves, dispersion relations, etc.
No waveguides

$\left.\begin{array}{l} \vec{k}\big|_{\text{along boundary}} \text{ is constant} \\ \\ \omega \text{ is constant} \end{array}\right\}$ Arrive from Symmetry

Symmetry $\longleftrightarrow$ Conserved quantities
Compare to Noether's Theorem
symmetries $\longleftrightarrow$ conservation laws

Hamiltonian Optics — relates to what we see in classical Hamiltonian formalism

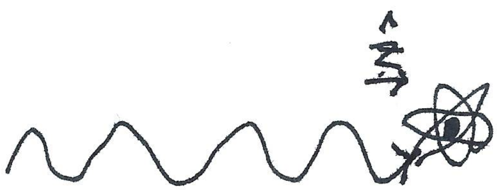
↓

WKB Theory — if mode has slowly varying particles then waves look like plane waves locally

Dispersive (insulating) medium

$\epsilon(\omega)$ $\mu(\omega)$ are functions of ω

dilute atomic/molecular gas — Clausius-Mossotti relation (Griffiths Ch 4)



sloshes electrons around

plane-polarized (in $\hat{x}$ direction)

non-relativistic motion

$$m\frac{d^2}{dt^2}x = f_e = f_{\text{binding}} + f_{\text{damping}} + f_{\text{drive due to wave}}$$

relative to equilibrium position

f_{binding} is complicated can approximate it as a SHO

$$m\ddot{x} = -m\omega_0^2 x - m\gamma\dot{x} + qE_x(x,t)$$

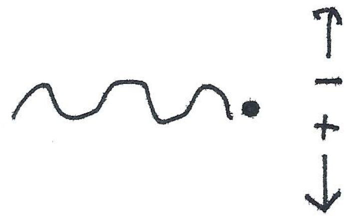
non-relativistic — small displacements — linear drag — non-relativistic

$$\vec{E} \sim (\hat{x} E_0 e^{-i\omega t} + cc)/2$$

Looking "~~stdy~~ steady-state" harmonic response

$$x \sim x_0 e^{-i\omega t}$$

$$x_0 = \frac{q/m}{\omega_0^2 - \omega^2 - i\gamma\omega} E_0$$

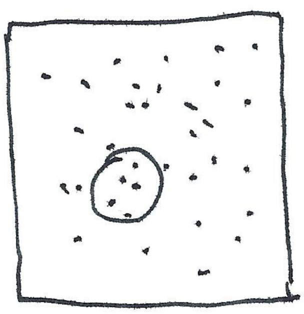


puts a periodic dipole moment

$$\vec{p}(t) = q x(t) = \frac{q^2/m}{\underset{\uparrow}{\omega_0^2} - \omega^2 - i\gamma\underset{\nwarrow}{\omega}} E_0 e^{-i\omega t}$$

drive frequency of wave

resonant frequency



λ

Average over lots of atoms

Each molecule has f_j electrons with resonant frequency ω_j and damping coefficient γ_j

$\frac{N}{V}$ molecules per unit volume

$$\vec{P} = \underbrace{\frac{Nq^2}{Vm} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j \omega}}_{\epsilon_0 \chi_e \leftarrow \text{susceptibility}} \vec{E}$$

$$\epsilon = \epsilon_0 (1 + \chi_e)$$

Oscillator strengths

$$\frac{\epsilon}{\epsilon_0} = 1 + \frac{Nq^2}{Vm\epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j \omega}$$

$\mu = \mu_0 \qquad \omega^2 = \epsilon \mu_0 k^2 \qquad v = \dfrac{1}{\sqrt{\epsilon \mu_0}}$ propagation and attenuation since ϵ is complex

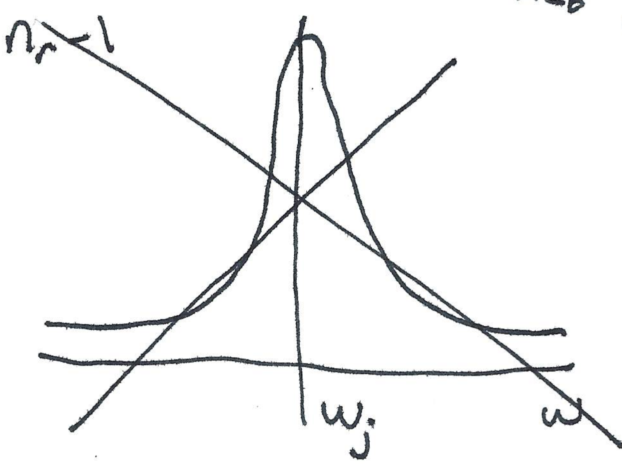
$k = k_r + i k_i$ for real, positive ω

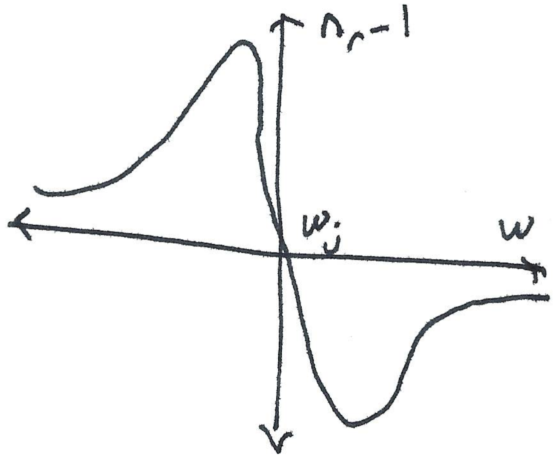
$= k_r + i\kappa$

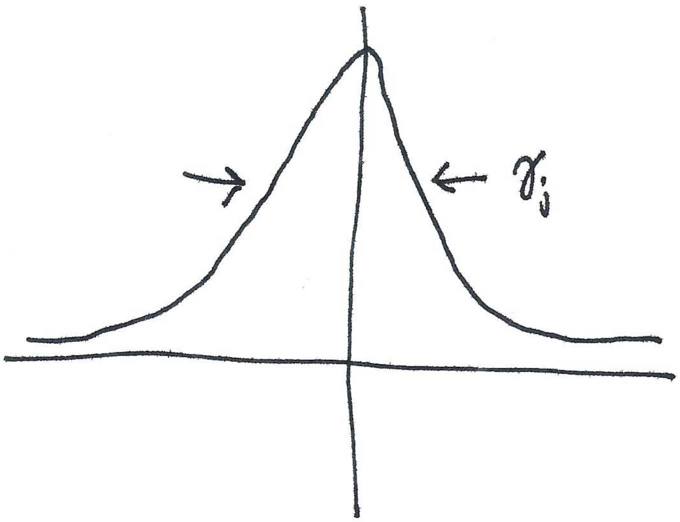
gas $\dfrac{\epsilon}{\epsilon_0} = 1 +$ something small   Taylor expansion

$$n_r = \frac{c k_r}{\omega} \approx 1 + \frac{N q^2}{V m \epsilon_0} \sum_j \frac{f_j(\omega_j^2 - \omega^2)}{(\omega_j^2 - \omega^2)^2 + \gamma_j^2 \omega^2}$$

Lorentzian shape × some factor



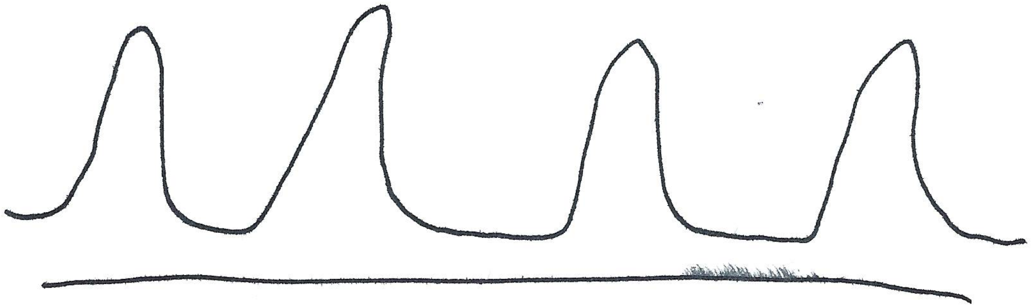




absorption coefficient

$$\underset{2\kappa}{\overset{2k_i}{\shortparallel}} = \frac{N q^2 \omega^2}{V m \epsilon_0 c} \sum \frac{f_j \gamma_j}{(\omega_j^2 - \omega^2)^2 + \gamma_j^2 \omega^2}$$

Lorentzian



Far away from resonances

$$n_r \approx 1 + \frac{Nq^2}{Vm\epsilon_0}\sum_j \frac{f_j}{\omega_j^2-\omega^2}$$

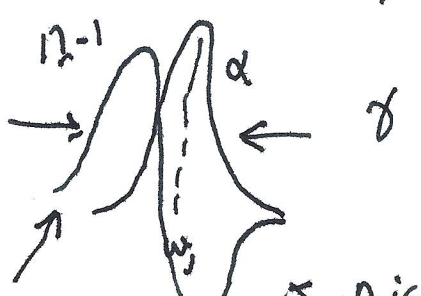
$\frac{1}{\omega_j^2-\omega^2}$ is not blowing up

$\omega \ll \omega_j$ (far below resonance)

$$n \cong 1 + \left(\frac{Nq^2}{Vm\epsilon_0}\sum_j \frac{f_j}{\omega_j^2}\right) + \omega^2\left(\frac{Nq^2}{2Vm\epsilon_0}\sum_j \frac{f_j}{\omega_j^4}\right) + \cdots$$

Taylor expand in ω^2

Cauchy Formula



n is increasing with increasing ω

n is increasing with increasing frequency ω

Anomalous Dispersion

n decreases with ω

Only happens near resonances

Often hidden due to large absorption

Most materials transparent to visible light

Resonances are UV

↑ absorption is low

UV resonance

$$\frac{\Delta n}{\Delta \omega} > 0$$

$n > 1$

Despite oversimplification this approximation does a surprisingly good job.