Phys 110B 09Oct20

Exam next week for 3 hours
Relativity and maybe dispersion
Open book, provided, not internet

"cylindrical waveguides"

$\vec{E}_\perp \propto \vec{\nabla}_\perp \psi \quad E_z \propto \psi \quad$ TM

$\vec{B}_\perp \propto \vec{\nabla}_\perp \psi \quad B_z \propto \psi \quad$ TE

only possible with 2 or more conductors → $\vec{E}_\perp \propto \vec{\nabla}_\perp \psi \quad E_z = 0,\ B_z = 0 \quad$ TEM

eg coaxial cable

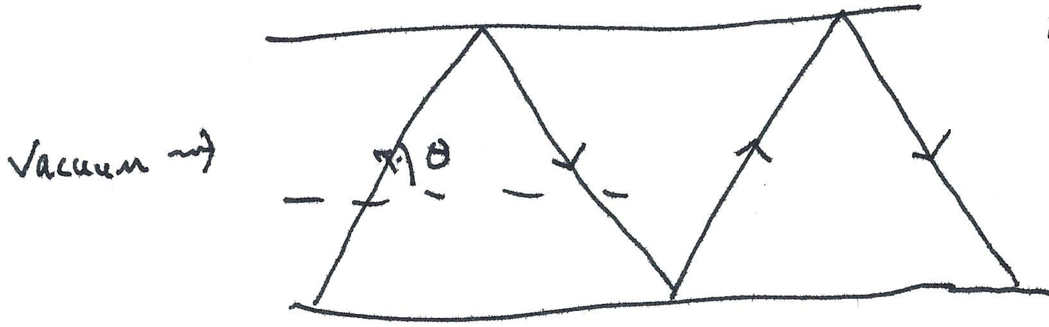
$$(\nabla^2 + k_\perp^2)\psi = 0 \qquad \psi = e^{ikz}\phi$$

$$\omega^2 = c^2 k_\perp^2 + c^2 k^2$$

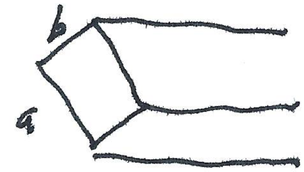
$$P = \tfrac{1}{2}\mathrm{Re}\int \vec{S}\cdot\hat{z}\,da \propto \int |\vec{\nabla}_\perp \psi|^2 da$$

$$P = \sum_{\text{modes}} P_j$$

modes are orthogonal



eg rectangular waveguide



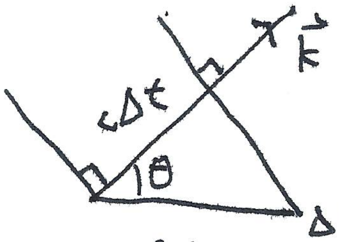
$$\omega^2 = c^2(k_x^2 + k_y^2 + k_z^2)$$

$$\vec{k} = \hat{x}\frac{\pi m}{a} + \hat{y}\frac{\pi n}{b} + k\hat{z}$$

$$\omega^2 = c|\vec{k}_{tot}|^2 = \sqrt{c^2k^2 + \underbrace{\frac{c^2\pi^2 m^2}{a^2} + \frac{c^2\pi^2 n^2}{b^2}}_{c^2k^2_{\perp mn}}}$$

$$\cos\theta = \frac{k}{|k_\perp|} = \sqrt{1 - \frac{c^2k_\perp^2}{\omega^2}}$$

$$v_g = c\cos\theta = c\sqrt{1 - \frac{c^2k_\perp^2}{\omega^2}} = \frac{d\omega}{dk} = c^2\frac{k}{\omega}$$



$$c\Delta t = \Delta z\cos\theta$$

$$\Longrightarrow \frac{\Delta z}{\Delta t} = \frac{c}{\cos\theta} = \frac{c}{\sqrt{1 - \frac{c^2k_\perp^2}{\omega^2}}}$$

circular

Bessel Functions

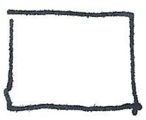
$\vec{E}(x,y,z)$ 6 functions of 3 coordinates each

$\vec{B}(x,y,z)$

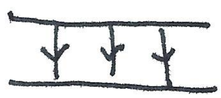
Solve Helmholtz for 1 scalar field function of 2 coordinates

TEM modes

2d cross-sectional boundary must not be simply bounded

  $-\nabla^2\phi = 0$

X X

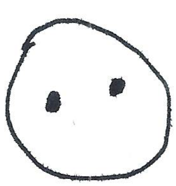
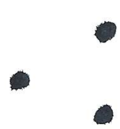
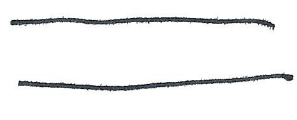
coaxial cable   2 wire pair  stripline 1 TEM Mode

✓ ✓ ✓

$\omega^2 = c^2k^2$ dispersionless in coaxial cable

$\frac{\omega}{k} = c$

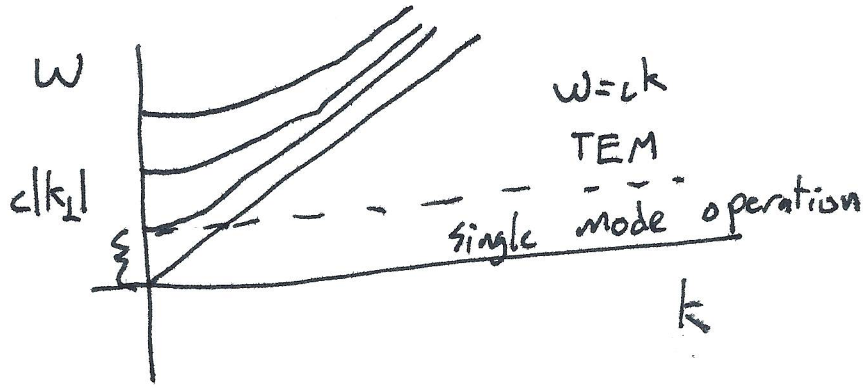
50Ω Characteristic Impedance

2 TEM ″ ″

TEM mode $\omega^2 = c^2 k^2$

TE mode $\omega^2 = c^2 k^2 + c^2 k_\perp^2$

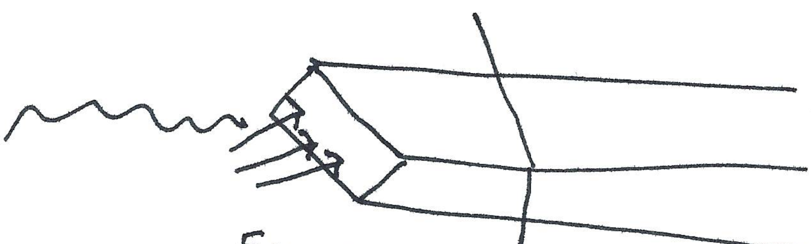


countably infinite modes

at given frequency, only finite number of modes

$$P = \sum_{\text{modes}} P_j$$

$$\frac{U}{\text{length}} = \sum_{\text{modes}}$$



$$\frac{\text{Energy}}{\text{length}} = \text{Finite}$$

convergent series $\rightarrow$ longitudinal Power

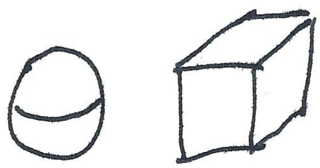
Energy / Length

$$(\nabla^2 + k^2)\psi = 0$$

$$-\nabla^2\psi = k^2\psi$$

Metallic Cavities

voids in conductors



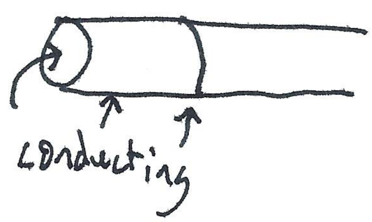
normal modes

conducting walls

all field components need to satisfy $(\nabla^2 + k^2)\Psi = 0$

with constraints $\vec{\nabla}\cdot\vec{E} = 0$, $\vec{\nabla}\cdot\vec{B} = 0$ B.C.'s

special case of cavities → "capped" waveguides



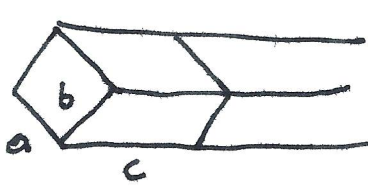
start with infinite waveguide

choose k to satisfy B.C. at the ends

k will be discrete

e.g. Rectangular cavities ("boxes")

2 possible Polarizations if $n_x n_y n_z = 0$



3 sines or cosines

$$E_x = E_1 \cos(k_x x)\sin(k_y y)\sin(k_z z)e^{-i\omega t}$$

$$E_y = E_2 \sin(k_x x)\cos(k_y y)\sin(k_z z)e^{-i\omega t}$$

$$E_z = E_3 \sin(k_x x)\cos(k_y y)\cos(k_z z)e^{-i\omega t}$$

$$k_x = \frac{n_x\pi}{a} \quad k_y = \frac{n_y\pi}{b} \quad k_z = \frac{n_z\pi}{c}$$

$$\frac{\omega^2}{c^2} = \pi^2\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right)$$

$\vec{\nabla}\times\vec{E} = i\omega\vec{B}$ automatically satisfied

n_x, n_y, n_z are non-negative integers at least 2 are non zero

$$k_x E_1 + k_y E_2 + k_z E_3 = 0$$