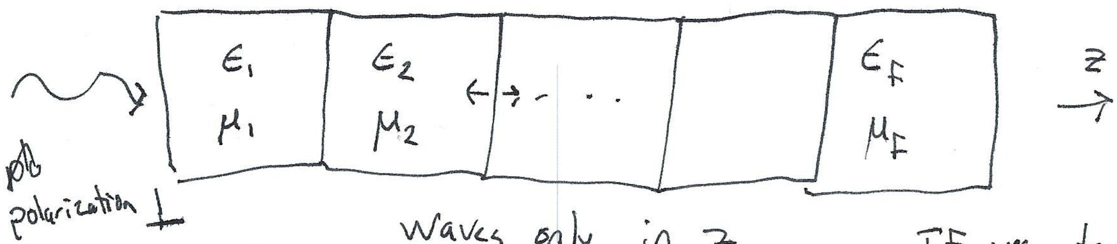
Phys 110B 02Oct20

Exam 15Oct20 Tentatively

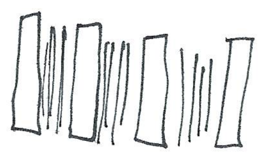
Transfer Matrices



Waves only in z

If we try to follow individual waves, it'll get messy with a large summation

e.g. dielectric mirror



"Drift" Matrix

"Drift" from 1 side of a homogeneous mode to another

$$\begin{bmatrix} A_r(z+L) \\ A_\ell(z+L) \end{bmatrix} = \underbrace{\begin{bmatrix} e^{ikL} & 0 \\ 0 & e^{-ikL} \end{bmatrix}}_{= D} \begin{bmatrix} A_r(z) \\ A_\ell(z) \end{bmatrix}$$

"Interface" Transfer matrix

jump across an interface

no surface charge
no current

$$\Rightarrow E_x(z^-) = E_x(z^+)$$

$$\frac{1}{\mu^-} B_y(z^-) = \frac{1}{\mu^+} B_y(z^+)$$

k | k'

μ | μ'

z

$$\begin{bmatrix} A_r(z^+) \\ A_\ell(z^+) \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{k'+k}{2k'} & \frac{k'-k}{2k'} \\ \frac{k'-k}{2k'} & \frac{k'+k}{2k'} \end{bmatrix}}_{= F} \begin{bmatrix} A_r(z^-) \\ A_\ell(z^-) \end{bmatrix}$$

$$\frac{k}{\sqrt{\mu\epsilon}} = \omega = \frac{k'}{\sqrt{\mu'\epsilon'}}$$

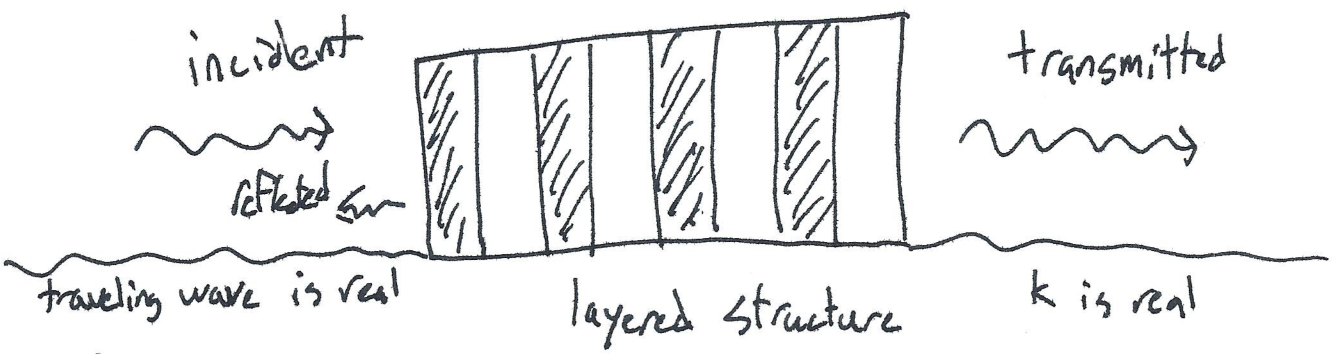
can eliminate k in favor of ϵ, μ

$$\begin{array}{c|c|c|c} \epsilon_1 & \epsilon_2 & & \epsilon_f \\ \mu_1 & \mu_2 & \cdots & \mu_f \end{array}$$

$$\begin{bmatrix} A_r(z_f) \\ A_\ell(z_f) \end{bmatrix} = \cdots F_3 D_3 F_2 D_2 F_1 D_1 \begin{bmatrix} A_r(z_1) \\ A_\ell(z_1) \end{bmatrix}$$

$$\vec{A} = \hat{x} A_r(z) e^{-i\omega t} + \hat{x} A_\ell(z) e^{-i\omega t} \qquad \vec{E} = -\frac{\partial}{\partial t}\vec{A}(z,t)$$

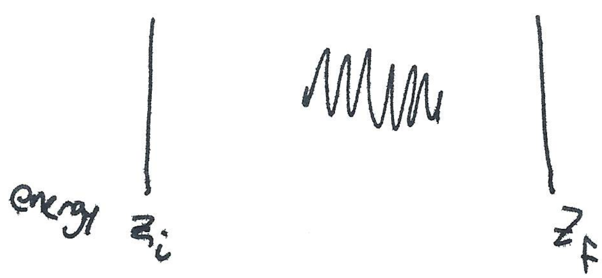
$$A_r(z) \propto e^{ikz} \qquad A_\ell(z) \propto e^{-ikz} \qquad \vec{B} = \hat{z} \times \frac{\partial}{\partial z}\vec{A}(z,t)$$



$$\langle \vec{S} \rangle = \frac{1}{2} \mathrm{Re} \left\langle \frac{\vec{E} \times \vec{B}^*}{2\mu^*} \right\rangle = \frac{k\omega}{2\mu} \hat{z} \left(|A_r(z)|^2 - |A_\ell(z)|^2 \right)$$

over 1 optical period

since steady state, there can be no energy build up



$$\frac{\omega k_i}{\mu_i}\left[A_r(z_i)^* \; A_\ell(z_i)^*\right] G \begin{bmatrix} A_r(z_i) \\ A_\ell(z_i) \end{bmatrix} = \frac{\omega k_f}{\mu_f}\left[A_r(z_f)^* \; A_\ell(z_f)^*\right] G \begin{bmatrix} A_r(z_f) \\ A_\ell(z_f) \end{bmatrix}$$

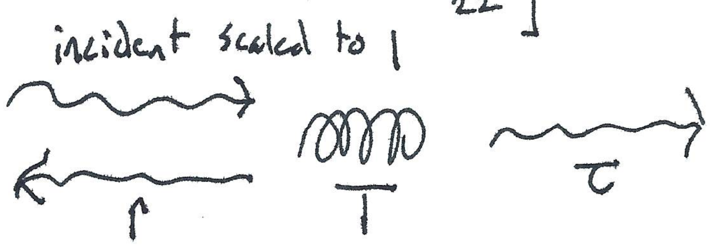
$$G = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

T = overall transfer matrix between z_i and z_f

$$\boxed{\frac{k_f}{\mu_f} T^\dagger G T = \frac{k}{\mu} G}$$ energy conservation

D satisfies
F satisfies $\Longrightarrow$ satisfied in general

$$\text{generic } T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$



$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{bmatrix} 1 \\ r \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix} = \begin{bmatrix} T_{11} + rT_{12} \\ T_{21} + rT_{22} \end{bmatrix}$$

$$r = -\frac{T_{21}}{T_{22}} \qquad \tau = \frac{T_{11}T_{22} - T_{12}T_{21}}{T_{22}} = \frac{\det T}{T_{22}}$$

(Power) Reflection Coefficient

$$R = \frac{\frac{\omega k_i}{2\mu_i}|r|^2}{\frac{\omega k_i}{2\mu_i}|1|^2} = |r|^2 = \left|\frac{T_{21}}{T_{22}}\right|^2 = \frac{|T_{21}|^2}{|T_{22}|^2}$$

Transmission Coefficient

$$T = \frac{\frac{\omega k_f}{2\mu_f}|\tau|^2}{\frac{\omega k_i}{2\mu_i}|1|^2} = \frac{k_f}{k_i}\frac{\mu_i}{\mu_f}\frac{|\det T|^2}{|T_{22}|^2}$$

$$|T_{11}|^2 = \frac{k_i}{k_f}\frac{\mu_f}{\mu_i} + |T_{21}|^2$$

$$|T_{22}|^2 = \frac{k_i}{k_f}\frac{\mu_f}{\mu_i} + |T_{12}|^2$$

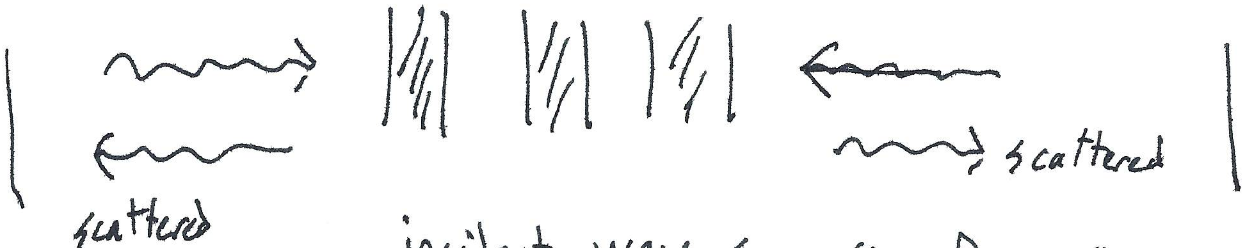
$$T_{12}T_{11}^* = T_{21}^*T_{22}$$

$$T + R = \frac{1}{|T_{22}|^2}\left(\frac{k_i}{k_f}\frac{\mu_f}{\mu_i} + |T_{21}|^2\right) = \frac{|T_{22}|^2}{|T_{22}|^2} = 1$$

$$T + R = 1$$

Scattering Matrix

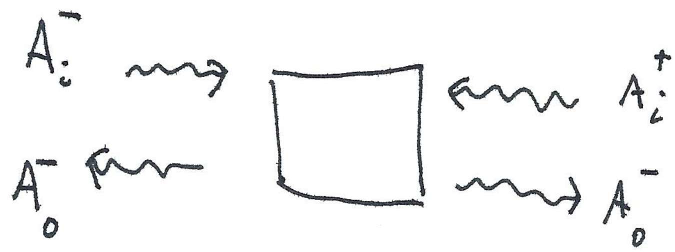
$$\begin{bmatrix} A_r(z') \\ A_l(z') \end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} A_r(z) \\ A_l(z) \end{bmatrix}$$



$z > 0$

incident wave can come from either side

$z < 0$



$$\begin{bmatrix} A_0^+ \\ A_i^+ \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} A_i^- \\ A_0^- \end{bmatrix}$$

$$\begin{bmatrix} A_0^- \\ A_0^+ \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{-T_{21}}{T_{22}} & \frac{1}{T_{12}} \\ \frac{\det T}{T_{22}} & \frac{T_{12}}{T_{22}} \end{bmatrix}}_{S \,=\, \text{Scattering Matrix}} \begin{bmatrix} A_i^- \\ A_i^+ \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \text{output} \\ \text{amplitudes} \end{bmatrix}}_{\text{scattered}} = S \underbrace{\begin{bmatrix} \text{input} \\ \text{amplitudes} \end{bmatrix}}_{\text{incident}}$$

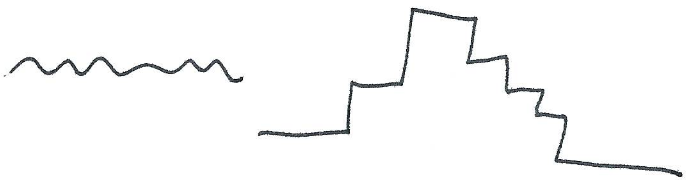
if $k_i = k_f$

$\mu_i = \mu_f$

Vacuum  Vacuum

S is unitary — can be generalized in many ways

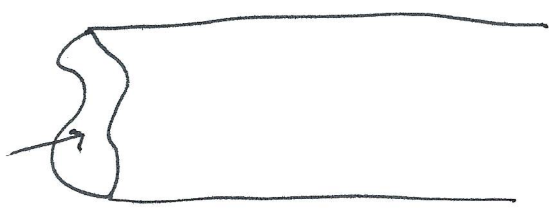
QM



dielectric mirror

Hollow metallic generalized cylindrical waveguides

hollow



perfectly conductive

fixed cross-sectional shape

"Pilot vector" approach

TE, TM, TEM modes