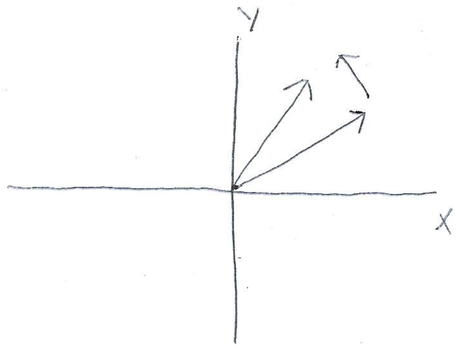
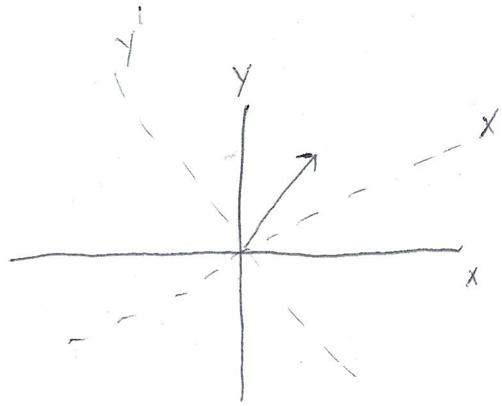
Phys 110B 31 Aug 20

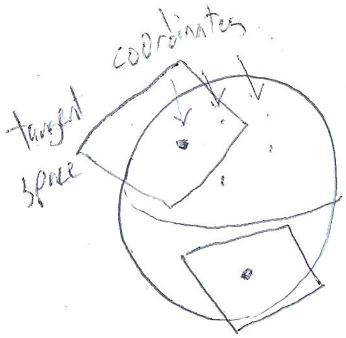


Active Transformation
"Alibi"



Passive Transformation
"Alias"

Distinguish manifold and vector on manifold



"Manifold"

~~A~~Every point has a different tangent space
Even if coordinate system isn't linear,
the transformations are linear

$$\begin{bmatrix} v'_x \\ v'_y \\ v'_z \end{bmatrix} = R(\phi)\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}, \quad R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Vectors
"Contravariant"

covectors
"covariant"

Tensor index notation and Einstein Summation Convention

$$v'^j = R^j{}_i v^i = \sum_i R^j{}_i v^i$$

$i, j, k, \ldots \in \{1,2,3\}$
x, y, z
r, θ, ϕ

$\mu, \nu, \lambda, \ldots \in \{0,1,2,3\}$
$t \quad x \quad y \quad z$
$r \quad \theta \quad \phi$

Einstein Summation Convention
unless otherwise noted, a repeated index (one up, other down) is implicitly summed over

$$R^j_{\ i} V^i = R^j_{\ k} V^k$$

j is free index, i, k are summed or dummy indices

Contravariant vector components

$$(x, y, z) \to (x', y', z')$$

$$x^i \to x'^i \quad \text{coordinate transformation}$$

Chain Rule $\to$

$$dx'^i = \cancel{dx} \frac{\partial x'^i}{\partial x^j} dx^j$$

Contraction: contracting vectors with 2 indices to 1 index

$\uparrow$ Jacobian Matrix — can vary, not necessary constant

contravariant transformation Rule

Covariant ~~vector components~~ for operator transformation

$$x^i \to x'^i$$

$$\frac{\partial}{\partial x^i} \to \frac{\partial}{\partial x'^i}$$

$$\frac{\partial}{\partial x^i} = \partial_i \qquad \frac{\partial}{\partial x'^i} = \partial'_i$$

inverse Jacobian

Chain Rule

$$\partial'_i = \frac{\partial}{\partial x'^i} = \frac{\partial x^j}{\partial x'^i} \frac{\partial}{\partial x^j} = \frac{\partial x^j}{\partial x'^i} \partial_j$$

Jacobian $\frac{\partial x'^i}{\partial x^j} = J^{i}_{\ j}$ — space matters

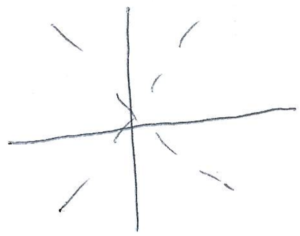
"co is down low"

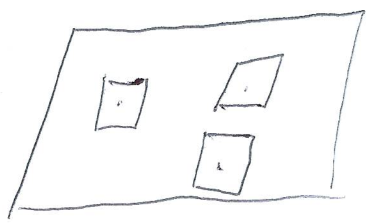
$$\begin{matrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{matrix} \quad x^\mu \qquad \begin{matrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{matrix} \quad x'^\nu \qquad dx^\mu$$

coordinates don't necessary transform like their indices suggest. They aren't tensors

Cartesian Coordinates

$$X'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} x^{\nu}$$ only true for Cartesian Coordinates





isomorphic in Special Relativity

Contravariant components dx^{μ} covariant components ∂_{μ}

dX

vectors

∇

co-vectors

Tensors are classified by transformation properties by the locations and #'s of free indices

Rank-1 dx^{μ} tensor

∂_{μ} Rank-1 tensor

Rank-$\begin{bmatrix}0\\1\end{bmatrix}$ tensor

Rank-$\begin{bmatrix}1\\0\end{bmatrix}$ tensor

Rank-2 tensor $R^{j}{}_{k}$

Rank-$\begin{bmatrix}1\\1\end{bmatrix}$ tensor

$$R'^{j}{}_{k} = \frac{\partial x^{l}}{\partial x'^{k}} \frac{\partial x'^{j}}{\partial x^{i}} R^{i}{}_{l}$$

$\downarrow$

$R'^{j}{}_{k}$

$\vec{V} = V^{i} \vec{b}_{i}$

$V_{i} \vec{b}^{i}$

$R^{j}{}_{k}$ R^{jk} R_{jk} $R_{j}{}^{k}$

R_{jkl} R_{ijkl}

Rank-0 tensor $a' = a$

scalar

~~not just number~~ not just a number doesn't matter coordinate systems

transforms correctly ~~under~~ ~~trans~~