Phys 110B 30Sep20

Simple "Drude" collision model of conductor

$$\sigma(\omega) = \frac{\frac{n_e e^2}{\xi}}{1 - i\frac{m\omega}{\xi}} = \frac{\sigma_0}{1 - i\sigma_0 \frac{m\omega^2}{n_e e^2}}$$

Dispersion relation inside conductor $\quad K^2 = \mu\epsilon\omega^2 + i\omega\mu\sigma$

$$k^2 = \omega^2\mu\left(\epsilon + i\frac{\sigma}{\omega}\right)$$

No way to tell the difference between free and bound charges since free charge oscillates due to fields

"typical metal" "large friction" $\frac{m\omega}{\xi} \ll 1$

or $\omega \ll \frac{n_e e^2}{\sigma_0 m}$

$\sigma(\omega) = \sigma_0$ constant

"good plasma" "small friction" $\frac{m\omega}{\xi} \gg 1$ or $\omega \gg \frac{n_e e^2}{\sigma_0 m}$

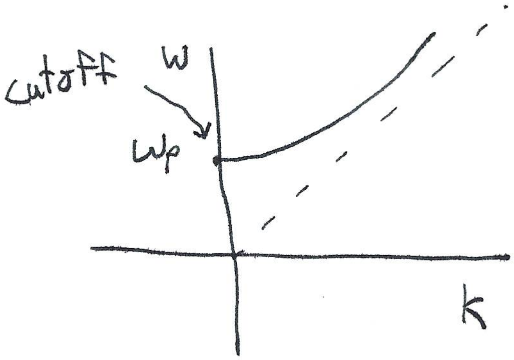
$$\sigma(\omega) \simeq i\frac{ne^2}{m\omega}$$

$$k^2 = \mu\epsilon\omega^2\left(1 - \frac{ne^2}{m\epsilon\omega^2}\right) = \mu\epsilon\omega^2\left(1 - \frac{\omega_p^2}{\omega}\right)$$

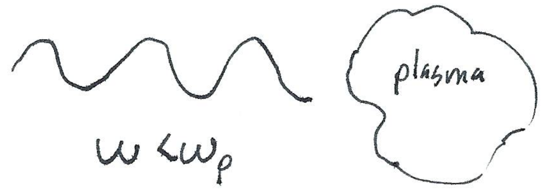
$\omega_p^2 = \frac{ne^2}{m\epsilon}$ = (electron) plasma frequency squared if $\mu = \mu_0$ $\epsilon = \epsilon_0$

Vacuum $\omega^2 = c^2k^2$

$\Rightarrow \omega^2 = \omega_p^2 + c^2k^2$



plasma cannot support a traveling EM wave below ω_p



evanescent

$(\hbar\omega)^2 = (\hbar\omega_p)^2 + c^2(\hbar k)^2$

$\mathcal{E}^2 = \underset{mc^2}{\text{const}}^2 + c^2p^2$ relativistic mass-energy relation

so far Linear Homogeneous Isotropic

now ~~[illegible]~~ (structured) (less symmetric) media

but locally look LHI

match up locally irreducible waves via boundary conditions

~~PD~~ Piecewise uniform dielectric

$n_0 \,|\, n_1 \quad |\, n_2 \quad |\, n_3 \quad |\, n_4 \,| \; \ldots$

$z_0 \qquad z_1 \qquad z_2 \qquad z_3 \qquad z_4$

parallel planar boundaries between regions of constant n

waves propagating transverse to boundaries

<u>Transfer Matrices</u>

Transfer ~~Matrices~~

dielectric boundaries no changes permeability

no surface charges or surface currents $(\mu = \mu_0)$

Monochromatic waves $e^{-i\omega t}$ time dependence

in any one region we still have $\omega^2 = \frac{c^2}{n} k^2$, and $\vec{k} = k\hat{z}$

n | n'

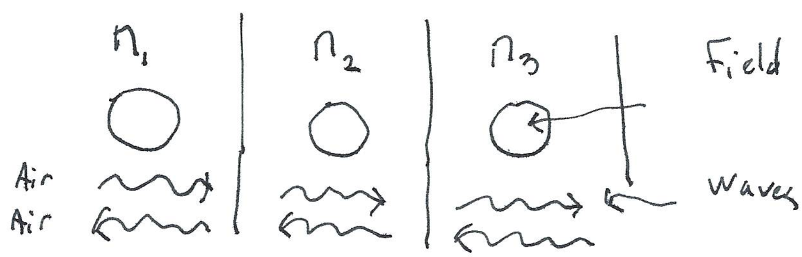
$E_x \sim -\partial_t A \sim i\omega A \Rightarrow A = A'$

$B_y \sim \partial_z A \sim ikA \Rightarrow kA = k'A'$

$\partial_z A = \partial_z A'$

$\vec{E}_{\parallel} = \vec{E}'_{\parallel} \qquad \vec{B}_{\parallel} = \vec{B}'_{\parallel}$

$\parallel$ parallel to boundary



"drift" waves drift through a region of fixed n, fixed k

$A_r(X+L) = e^{ikL} A_r(x) \qquad k>0$

$A_\ell(X+L) = e^{-ikL} A_\ell(x)$

$$\begin{bmatrix} A_r(x+L) \\ A_\ell(x+L) \end{bmatrix} = \begin{bmatrix} e^{ikL} & 0 \\ 0 & e^{-ikL} \end{bmatrix} \begin{bmatrix} A_r(x) \\ A_\ell(x) \end{bmatrix}$$

X_0

$X_0 + L_1 = X_1$

Interface (across a boundary)

$$A_r(x^-) + A_\ell(x^-) = A_r(x^+) + A_\ell(x^+)$$

$$ikA_r(x^-) - ikA_\ell(x^-) = ik'A_r(x^+) - ik'A_\ell(x^+)$$

$$\begin{bmatrix} 1 & 1 \\ ik & -ik \end{bmatrix}\begin{bmatrix} A_r(x^-) \\ A_\ell(x^-) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ ik' & -ik' \end{bmatrix}\begin{bmatrix} A_r(x^+) \\ A_\ell(x^+) \end{bmatrix}$$

$$\begin{bmatrix} A_r(x^+) \\ A_\ell(x^+) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ ik' & -ik' \end{bmatrix}^{-1}\begin{bmatrix} 1 & 1 \\ ik & -ik \end{bmatrix}\begin{bmatrix} A_r(x^-) \\ A_\ell(x^-) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{k'+k}{2k'} & \frac{k'+k}{2k'} \\ \frac{k'-k}{2k'} & \frac{k'+k}{2k'} \end{bmatrix}\begin{bmatrix} A_r(x^-) \\ A_\ell(x^-) \end{bmatrix}$$

$$\frac{k}{k'} = \frac{\omega/v}{\omega/v'} = \frac{v'}{v} = \frac{n}{n'} = \sqrt{\frac{\epsilon}{\epsilon'}}$$

$n_0 \;|\; n_1 \;|\; n_2 \;|\; n_3 \;|\; n_4$

Interface

Drift

$$\begin{bmatrix} A_r(x_3^+) \\ A_\ell(x_3^+) \end{bmatrix} = T_{23} D_{23} T_{12} D_{12} T_{01} D_{01} \begin{bmatrix} A_r(x_0^-) \\ A_\ell(x_0^-) \end{bmatrix}$$

Matrices accumulate from right to left

Energy + Power transmission

no power
steady state

net power flux should be zero.