Phys 110B 16 Sep 20

$$\underset{\sim}{P} = \begin{bmatrix} E/c \\ \vec{p} \end{bmatrix} = \begin{bmatrix} \gamma m c \\ \gamma \vec{\beta} m c \end{bmatrix}$$

$$\frac{d}{dt}\vec{p} = \vec{F} \longleftarrow \text{3-force}$$

$\uparrow$ 3-momentum

$\underset{\sim}{k}$ = 4-wave vector

$\underset{\sim}{K}$ = 4-force

$$\frac{d}{d\tau} P^\mu = K^\mu \longleftarrow \text{4-force}$$

Minkowski force

$$\frac{dt}{d\tau}\frac{d}{dt}\vec{p} = \gamma \vec{F} \qquad \underset{\sim}{K} = \begin{bmatrix} ? \\ \gamma \vec{F} \end{bmatrix}$$

$$E = \sqrt{c^2 |\vec{p}|^2 + m^2 c^4}$$

$$\frac{d}{dt}E = \frac{1}{2}\left(\frac{1}{E}\right)\left(2c^2 \vec{p} \cdot \frac{d\vec{p}}{dt}\right) = c\,\frac{c\vec{p}}{E} \cdot \vec{F} = \vec{v} \cdot \vec{F}$$

$\downarrow \vec{\beta}$

$$\frac{dt}{d\tau} = \gamma \qquad \frac{d}{d\tau} P^\mu = K^\mu \qquad \underset{\sim}{K} = \begin{bmatrix} \gamma \vec{F} \cdot \vec{v}/c \\ \gamma \vec{F} \end{bmatrix}$$

$$\frac{d}{d\tau} m U^\mu = m \frac{d}{d\tau} U^\mu = m A^\mu \qquad m\underset{\sim}{A} = \underset{\sim}{K}$$

$$\underset{\sim}{A} \cdot \underset{\sim}{U} = 0 \;?$$

$$m\underset{\sim}{a} \cdot \underset{\sim}{U} = \underset{\sim}{K} \cdot \underset{\sim}{U} = \begin{bmatrix} \gamma \vec{F} \cdot \vec{v}/c \\ \gamma \vec{F} \end{bmatrix} \cdot \begin{bmatrix} \gamma c \\ \gamma \vec{\beta} c \end{bmatrix}$$

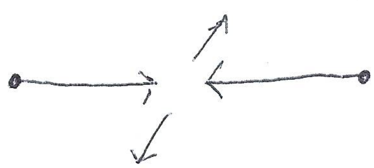
$$= -\left(\gamma \vec{F} \cdot \frac{\vec{v}}{c}\right)(\gamma c) + (\gamma \vec{F}) \cdot (\gamma \vec{v}) = -\gamma^2 \vec{F} \cdot \vec{v} + \gamma^2 \vec{F} \cdot \vec{v} = 0$$

$$\frac{d}{dt}\vec{p} = \vec{f} \qquad \frac{d}{d\tau}p^\mu = K^\mu$$

$$\underset{\sim}{P} = \begin{bmatrix} E/c \\ \vec{p} \end{bmatrix} = \begin{bmatrix} \gamma mc \\ \gamma\vec{\beta} mc \end{bmatrix} \qquad \underset{\sim}{K} = \begin{bmatrix} \gamma\vec{f}\cdot\vec{v}/c \\ \gamma\vec{f} \end{bmatrix}$$

Multiple particles makes τ complicated because it makes a lot of proper times

Relativistic "Collisions" (Impulsive)



Conservation Laws

$$P^\mu_{before} = P^\mu_{after} \qquad P^\mu = \sum_i P^\mu_i$$

i ← particle index

$\sqrt{-P^\mu P_\mu}$ is invariant mass

SR

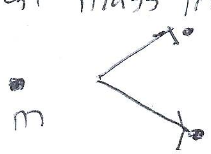
Covariance ← invariance of the content of laws of physics

"manifest" covariance → invariance of form

Invariance / invariant quantities → Lorentz scalars. Same value in all inertial frames

Conservation / conserved quantities → Same value at different times (in any one inertial frame) If it is conserved in 1 frame it is conserved in all frames but can hold different values

invariant rest mass m — Mass isn't conserved in decays

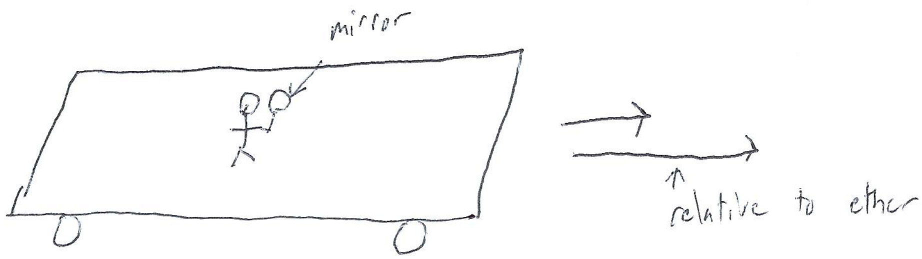


P^μ_{tot} conserved but not invariant

Impulsive "collisions"

Newtonian		SR	
Conservation of mass, momentum		Conservation of Energy, momentum	
explosive / inelastic	KE increase	KE increasing	mass decrease
elastic	stay the same	KE is conserved	mass conserved
sticky / inelastic	decrease	KE decreases	mass increases

Electrodynamics (Relativistic)



see reflection vanish if train goes at c

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\epsilon_0}\rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}\vec{E}$$

$$t \to t' = t$$
$$\vec{x} \to \vec{x} + \vec{v}t$$

This doesn't work for Galilean Transformations

Local Charge Conservation:

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0\vec{\nabla}\cdot\vec{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}\underbrace{\vec{\nabla}\cdot\vec{E}}_{=\frac{1}{\epsilon_0}\rho} = 0$$

$$\frac{\partial}{\partial x^i} = \partial_i$$

$$\partial_0 = \frac{\partial}{\partial x^0} = \frac{\partial}{c\,\partial t}$$

$$0 = \mu_0\left(\vec{\nabla}\cdot\vec{J} + \frac{\partial}{\partial t}\rho\right) = 0$$

$$\frac{1}{c}\partial_t c\rho + \vec{\nabla}\cdot\vec{J} = 0$$

$$\partial'_\mu J'^\mu = 0$$

$$\partial_\mu J^\mu = 0 \qquad \underset{\sim}{J} = \begin{bmatrix} c\rho \\ \vec{J} \end{bmatrix}$$

4-current

Work with EM potentials scalar potential Φ vector potential $\vec{A}$

Helmholtz decompositions

$$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \vec{B} = \vec{\nabla}\times\vec{A}$$ ← vector potential

generates magnetic field via derivatives

$$\vec{\nabla}\times\vec{E} + \frac{\partial}{\partial t}\vec{\nabla}\times\vec{A} = 0$$

$$\vec{\nabla}\times\left(\vec{E} + \frac{\partial}{\partial t}\vec{A}\right) = 0 \Rightarrow \vec{E} + \frac{\partial}{\partial t}\vec{A} = -\nabla\Phi$$ ← scalar potential

not necessarily a Lorentz scalar but in terms of vector analysis

$$\vec{E} = -\nabla\Phi - \frac{\partial}{\partial t}\vec{A}$$