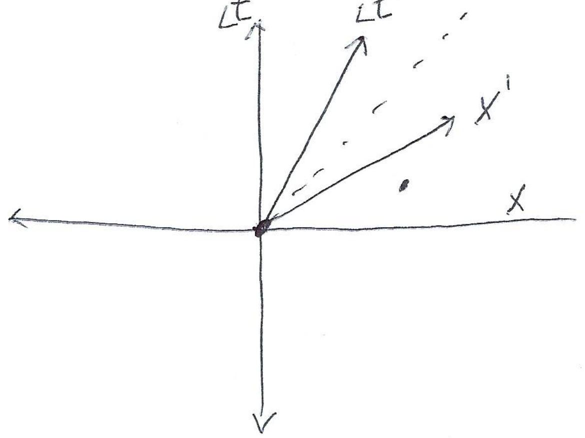
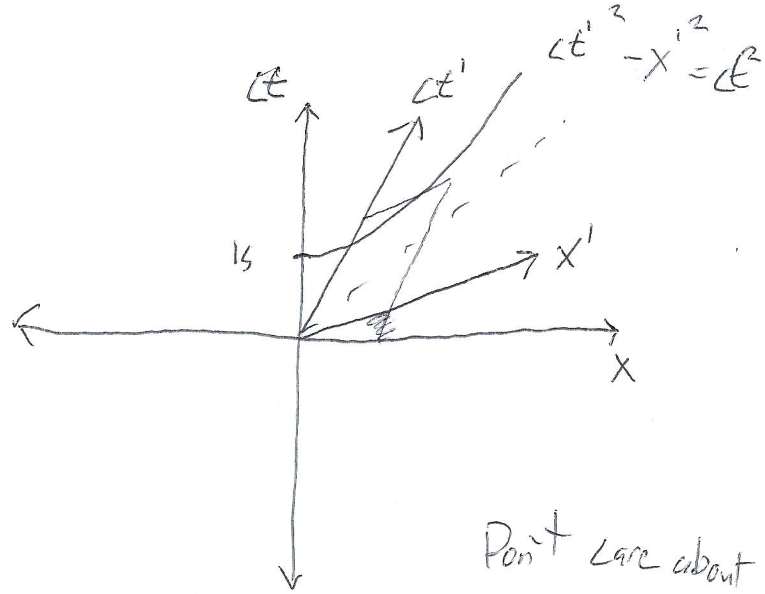
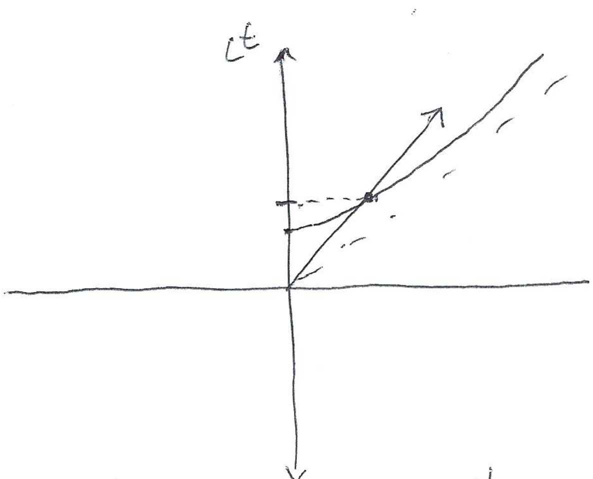
Phys 110B 14 Sep 20

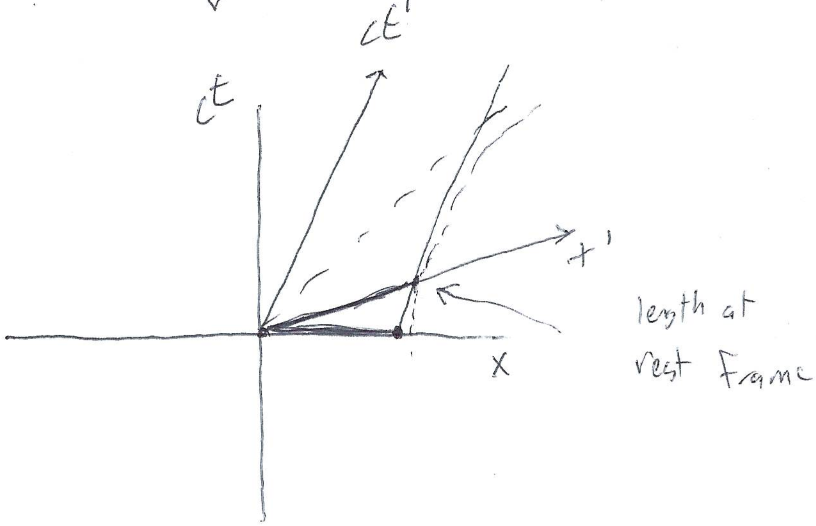
Minkowski Spacetime Diagrams





Don't care about the geometry of the page
Not Euclidean





$\vec{X}$ $\qquad$ dX^μ is a contravariant 4-vector

$\vec{V} = \frac{d\vec{x}}{dt}$ $\qquad$ ds^2 is invariant $\Rightarrow$ $d\tau$ is invariant

$\vec{A} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$ $\qquad$ $\frac{dX^\mu}{d\tau}$ is a 4-vector $\quad$ 4-velocity

"proper velocity"

bad notation

$$\frac{dX^\mu}{d\tau} = \frac{dt}{d\tau}\frac{dX^\mu}{dt} = \gamma\frac{dX^\mu}{dt}$$

$$d\tau = \frac{dt}{\gamma} \qquad \underset{\sim}{u} = \begin{bmatrix} \gamma c \\ \gamma \vec{v} \end{bmatrix}$$

$$\underset{\sim}{U} \cdot \underset{\sim}{U} = \frac{dX^\mu}{d\tau}\frac{dX_\mu}{d\tau} = \frac{dX^\mu}{d\tau}\eta_{\mu\nu}\frac{dX^\nu}{d\tau}$$

$$\frac{d\underset{\sim}{X} \cdot d\underset{\sim}{X}}{(d\tau)^2} = \frac{ds^2}{(d\tau)^2} = -\frac{(d\tau)^2}{(d\tau)^2} = -1$$

material particle $(m>0)$ $\quad \boxed{\underset{\sim}{U} \cdot \underset{\sim}{U} = -1}$ $\qquad$ A^μ mean something else

4-acceleration: $\frac{d}{d\tau}U^\mu = \frac{d^2}{d\tau^2}X^\mu = A^\mu$

$$\underset{\sim}{U} \cdot \underset{\sim}{A} = 0$$

$$\underset{\sim}{U} \cdot \underset{\sim}{U} = -1 \qquad U^\mu U_\mu = -1$$

$$\frac{d}{d\tau}(U^\mu U_\mu) = \frac{d}{d\tau}(-1) = 0$$

$$U^\mu \frac{d}{d\tau}U_\mu + \frac{dU^\mu}{d\tau}U_\mu = U^\mu a_\mu + a^\mu U_\mu = 2U^\mu a_\mu = 0 \Rightarrow U^\mu a_\mu = 0$$

$\underset{\sim}{a} \cdot \underset{\sim}{a} = ?$ $\sqrt{a_\mu a^\mu} =$ magnitude of proper acceleration

"g-Force" in the instantaneous rest frame

waves?

ω angular frequency

$\vec{k}$ 3-wave vector

$\omega = \omega(\vec{k})$

$\uparrow$ dispersion relation

wave $Ae^{i\theta(x,t)}$

$\theta(x,t) = \vec{k}\cdot\vec{x} - \omega t$

phase: a number of peaks

θ must be invariant

$$\theta = \underset{\sim}{K} \cdot \underset{\sim}{X} = -\frac{\omega}{c} ct + \vec{k}\cdot\vec{x} = -\omega t + \vec{k}\cdot\vec{x}$$

$$\underset{\sim}{K} \to \begin{bmatrix} \omega/c \\ \vec{k} \end{bmatrix} \qquad \underset{\sim}{X} \to \begin{bmatrix} ct \\ \vec{x} \end{bmatrix}$$

4-wavevector

$K^\mu \quad X^\mu \qquad \underset{\sim}{K}' = \Lambda \underset{\sim}{K}$ relativistic doppler shifts "headlighting effects"

$\theta = K_\mu X^\mu$

Planck-Einstein relations

$\cancel{E = \hbar\vec{k} \quad \vec{p} = \hbar}$

$E = \hbar\omega \qquad \vec{p} = \hbar\vec{k} \propto \vec{v}$

$\hbar K^\mu = P^\mu$ energy - momentum 4-vector

$$\underset{\sim}{P} = \begin{bmatrix} E/c \\ \vec{p} \end{bmatrix} = \begin{bmatrix} \frac{mc^2\gamma}{c} \\ mc\gamma\vec{\beta} \end{bmatrix}$$

$E = \gamma mc^2$ = relativistic energy

$\vec{p} = mc\gamma\vec{\beta} = \gamma m\vec{v}$ = relativistic 3-momentum

1. covariant (transform nicely)
2. resolve to Newton in the low-velocity limit
3. Preserve conservation laws

taylor expand $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$|\vec{v}| \ll c$ then $\vec{p} = m\vec{v} + \ldots$

$$E = \underset{\substack{\uparrow \\ \text{rest mass energy}}}{mc^2} + \frac{1}{2}m|\vec{v}|^2 + \ldots$$

relativistic kinetic energy $T = E - mc^2 = (\gamma - 1)mc^2$

$$P^\mu = m\underset{\substack{\uparrow \\ \text{4-vector}}}{U^\mu} = m\frac{d}{d\tau}X^\mu$$

m = rest mass * is a lorentz scalar/invariant

"names" the kind of particle, same with "q"

$\left.\begin{array}{l} \gamma m \text{ relativistic mass} \\ \vec{p} = \gamma m\vec{v} \end{array}\right\}$ Not a good approach

$$\underset{\sim}{P} \cdot \underset{\sim}{P} = -\frac{E^2}{c^2} + |\vec{p}|^2$$

$$m\underset{\sim}{U} \cdot (m\underset{\sim}{U})$$

$$m^2(-1)$$

$$\boxed{E^2 = c^2|\vec{p}|^2 + m^2c^4}$$

$$E = \gamma mc^2$$

$$\vec{p} = \gamma mc\vec{\beta}$$

$$\frac{c\vec{p}}{E} = \frac{\gamma mc^2\vec{\beta}}{\gamma mc^2} = \boxed{\vec{\beta} = \frac{c\vec{p}}{E}}$$

dynamics?

Newton's 2nd Law

$$\vec{F} = m\vec{a} = \frac{d}{dt}\vec{p}$$

$$\boxed{\frac{d}{dt}\vec{p} = \vec{F}} \leftarrow \text{3-force}$$

$$\vec{p} = \gamma m\vec{v}$$

$$\frac{d}{dt}\vec{p} = \vec{F}$$

$$\frac{d}{d\tau}p^\mu = \cancel{K}\,F^\mu$$

4-vector

4-Force

Minkowski Force