Phys 110B 11 Sep 20

Kinematic Consequences of Lorentz Transformations

time dilation "moving clocks run slow"

$\Delta y' = \Delta z' = 0$

$\Delta x' = 0$

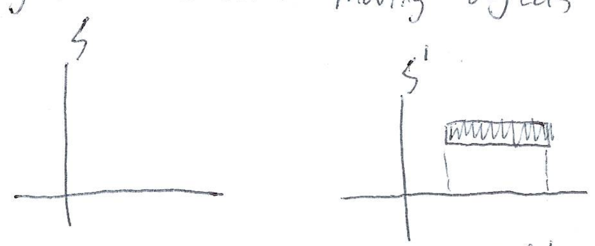
$\Delta t' = \Delta\tau$ = proper time interval

"property"

$$\begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix} = \begin{bmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{bmatrix} \begin{bmatrix} c\Delta t' \\ 0 \end{bmatrix}$$

$$\Rightarrow c\Delta t \Rightarrow \gamma c\Delta t' \quad \Rightarrow \Delta\tau = \Delta t' = \frac{\Delta t}{\gamma}$$

Length Contraction: moving objects are measured to be shorter



$\Delta t = 0$

$\Delta x = ?$

$\Delta t' = 0$

$\Delta x' = L'$ = proper length

looked at in same frame

$$\begin{bmatrix} c\Delta t = 0 \\ \Delta x \end{bmatrix} = \begin{bmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{bmatrix} \begin{bmatrix} c\Delta t' \\ L' \end{bmatrix}$$

can't both be at $t = 0$

$$\gamma c\Delta t' + \beta\gamma L' = 0$$

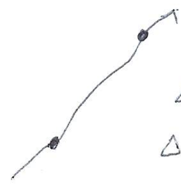
$$\Delta x = \beta\gamma c\Delta t' + \gamma L' = \beta\gamma(-\beta L') + \gamma L' = \gamma(1-\beta^2)L' = \frac{1}{\gamma}L'$$

Relativity of Simultaneity

$\Delta t' = 0$

$$\begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix} = \begin{bmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{bmatrix} \begin{bmatrix} 0 \\ \Delta x' \end{bmatrix} \Rightarrow \Delta t = \beta\gamma \frac{\Delta x'}{c}$$

Addition of velocity (1D)



$$\begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix} = \begin{bmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{bmatrix} \begin{bmatrix} c\Delta t' \\ \Delta x' \end{bmatrix}$$

particle velocity in S'

fixed velocity

$$\frac{\Delta x}{c\Delta t} = \frac{\gamma \Delta x' + \beta\gamma c\Delta t'}{\gamma c\Delta t' + \beta\gamma\Delta x'} = \frac{\frac{\Delta x}{c\Delta t'} + \beta}{1 + \beta \frac{\Delta x'}{c\Delta t'}}$$

Spacetime Intervals and Spacetime Geometry

$$ds^2 = -c^2 dt^2 + |d\vec{x}|^2 = -c^2 |dt'|^2 + |d\vec{x}|^2$$

$\uparrow$

spacetime interval

Lorentz Invariant

Same value in all inertial frames

$$ds^2 = dx^\mu \eta_{\mu\nu} dx^\nu = dX \cdot dX$$

$\uparrow$ Minkowski metric

$$ds^2 = -c^2 dt^2 + |d\vec{x}|^2 = -c^2 dt^2 + v^2 dt^2 = -c^2\left(1 - \frac{v^2}{c^2}\right) dt^2$$

$$= -c^2(1-\beta^2)\, dt^2 = -c^2 \frac{dt^2}{\gamma^2} = -c^2 d\tau^2$$

$\uparrow$

proper time interval

Δs^2 • numerical sign same for all observers

• Δs^2

< 0 in 1 frame $\Rightarrow < 0$ in all frames

time-like interval (time component dominates)

These points can be connected by a subluminal trajectory

> 0 in 1 frame $\Rightarrow > 0$ in all frames

spacelike interval/separation

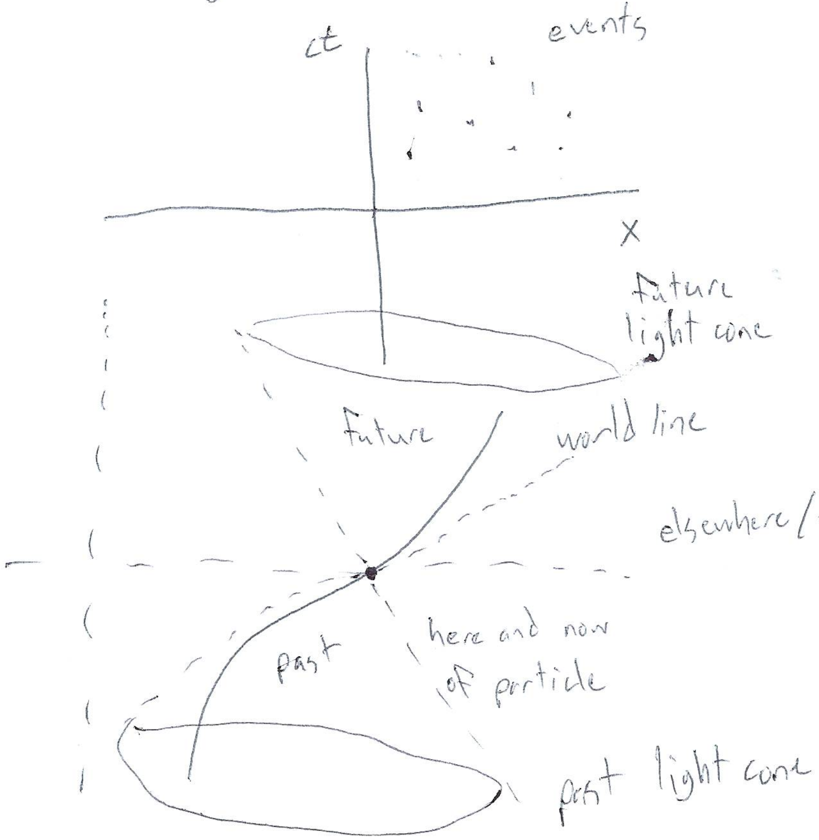
cannot be connected by anything travelling at c or less.

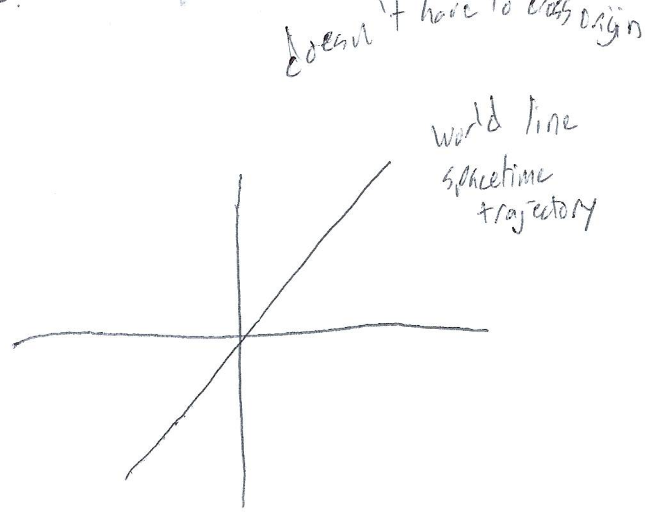
$= 0$

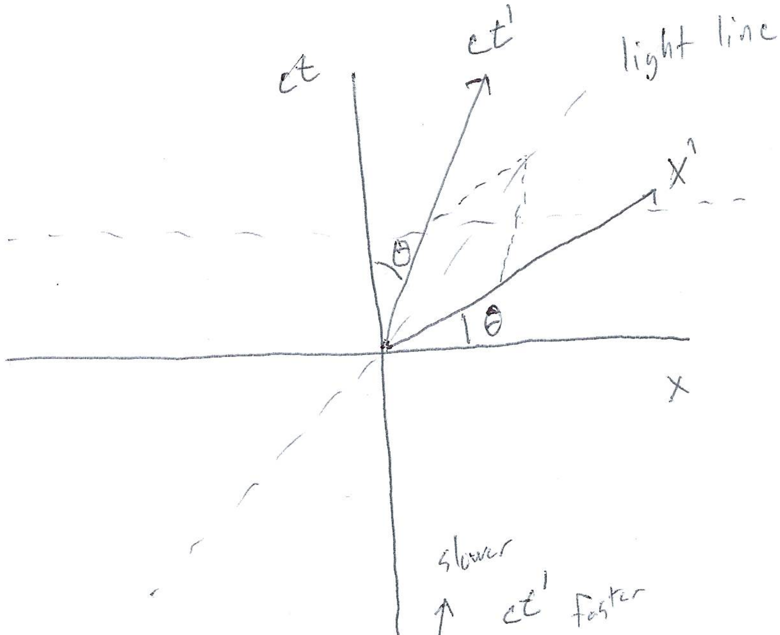
lightlike separation

could be connected by a light pulse.

Spacetime diagrams (Minkowski Diagrams)

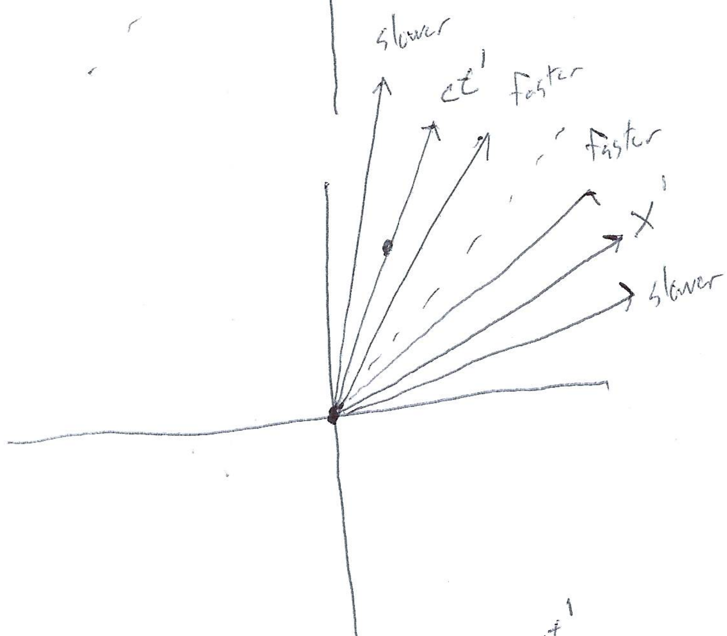




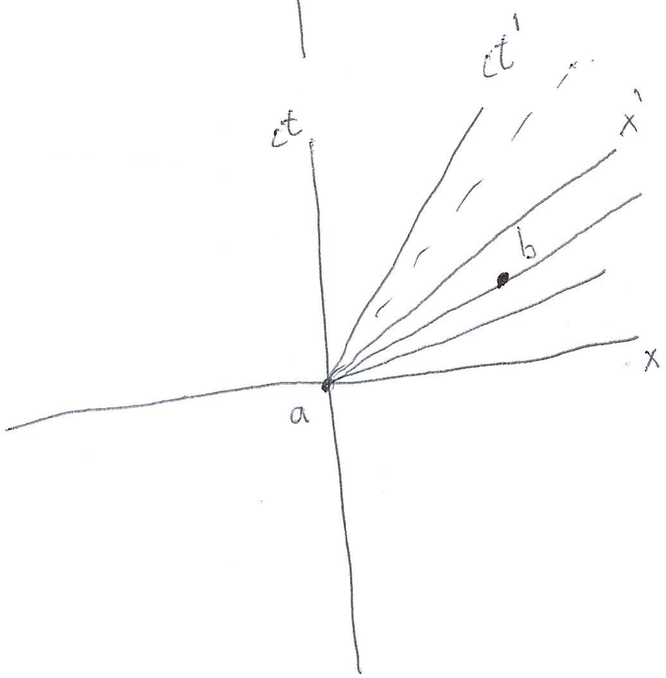


constant time

$\tan(\theta) = \beta$



spacial ordering changed



temporal ordering changed