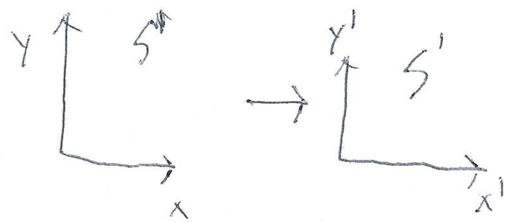
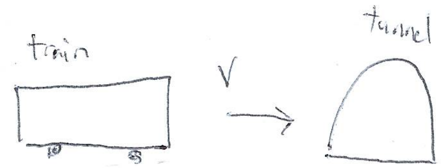
Phys 110B 09Sep20

Perpendicular Loretz transformation



$\perp$ don't transform at all



train just fits in tunnel
if train shrinks it fits in tunnel
if tunnel contracts, train won't fit
There can't be a disagreement in overall type of even

So ~~x'~~ $y' = y$, $z' = z$ under a boost

"Boost" in the x direction

$$X' = \begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \underbrace{\begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\Lambda} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} = X$$

$\beta = \frac{v}{c}$ = normalized velocity

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}}$ = Lorentz factor

"gamma" factor

$X' = \Lambda X$

"rapidity" formulation

$\beta = \tanh \xi$ — "rapidity" boost parameter

$\gamma = \cosh \xi$

$\beta\gamma = \sinh \xi$

ξ = "hyperbolic" angle
function of how much you're boosting

$X = \begin{bmatrix} ct \\ \vec{x} \end{bmatrix}^T$ = 4-vector

3 position

rescaled (?) time

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh\xi & -\sinh\xi \\ -\sinh\xi & \cosh\xi \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

"hyperbolic" rotation

$dX' = \cancel{A dX}\ \Lambda dX$

$X'^{\mu} = \Lambda^{\mu}{}_{\nu} X^{\nu}$

$X^0 = ct$
$X^1 = x$
$X^2 = y$
$X^3 = z$

$\vec{x}$ = 3-position
$\underset{\sim}{X}$ = spacetime position
$\vec{p}$ = 3-momentum
$\underset{\sim}{P}$ = 4-momentum

lower case sometime for 3-vector
and uppercase for 4-vector (not always)

All Lorentz transformations (boosts, rotations, translations, reflections etc)
Don't change the physics

$$c^2 dt^2 - |d\vec{x}|^2 = dx^{0^2} - (dx^{1^2} + dx^{2^2} + dx^{3^2})$$

$\begin{bmatrix} c(t+dt) \\ \vec{x}+d\vec{x} \end{bmatrix}$

$$= dx'^{0^2} - (dx'^{1^2} + dx'^{2^2} + dx'^{3^2})$$

$\begin{bmatrix} ct \\ \vec{x} \end{bmatrix}$

$$= c^2 dt'^2 - |d\vec{x}'|^2$$

Possible for all Lorentz transformations

Suppose events are connected by a light signal

$$c^2 |dt|^2 - |d\vec{x}|^2 = 0 = c^2 |dt'|^2 - |d\vec{x}'|^2$$

$$ds^2 = \text{spacetime interval} = c^2 dt^2 - |d\vec{x}|^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu}$$

$$\eta = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Lorentz transformations → all affine transformations
that $\eta_{\mu\nu} dx^{\mu} dx^{\nu} = \eta_{\mu\nu} dx'^{\mu} dx'^{\nu}$

ds^2 = Lorentz scalar

If transformation preserves this interval then it preserves the inner product

$$X^\mu \eta_{\mu\nu} Y^\nu \equiv X \cdot Y = X^0 Y^0 - \vec{x} \cdot \vec{y}$$

Lorentz inner product

Euclidian inner product

if $\underset{\sim}{X} \cdot \underset{\sim}{X} = \underset{\sim}{X}' \cdot \underset{\sim}{X}' \ \forall \underset{\sim}{X}$ then

$$\underset{\sim}{X} \cdot \underset{\sim}{Y} = \underset{\sim}{X}' \cdot \underset{\sim}{Y}' \ \forall X, Y$$

$$\underset{\sim}{X}' = \Lambda \underset{\sim}{X} \qquad \underset{\sim}{Y}' = \Lambda \underset{\sim}{Y}$$

$$\underset{\sim}{X} \cdot \underset{\sim}{Y} = \underset{\sim}{X}^T \eta \underset{\sim}{Y}$$

$$\underset{\sim}{X}' \cdot \underset{\sim}{Y}' = \underset{\sim}{X}^T \Lambda^T \eta \Lambda Y \Rightarrow \boxed{\Lambda^T \eta \Lambda = \eta}$$

defines the Lorentz "Group" of transformations

"Group" set of Lorentz transformations form a group

set of transforms represent a symmetry

includes: identity "do nothing"

Composition concatenating transforms is invariant

Associativity

Inverses

Orthochronous Lorentz Group

linear transformations

restricted (proper) Lorentz group

affine transformations

$$\underset{\sim}{X}' = \Lambda \underset{\sim}{X} + \underset{\sim}{X}_0$$

homogeneous Lorentz Group

Spatial Rotations (3 generators)

spatial inversion (parity) (1 generator)

Time reversal (1 generator)

Lorentz Boosts (3 generators)

Space Translations (3 generators)

Time translation (1 generator)

inhomogeneous Lorentz Group

Poincaré group