Phys 110B 04 Sep 20

## Orthonormal basis vectors

orthogonal and normalized

orthonormality:

$$\hat{e}_{\hat{i}} \cdot \hat{e}_{\hat{j}} = \left\{ \begin{matrix} 0 & \text{if } \hat{i} \neq \hat{j} \\ 1 & \text{if } \hat{i} = \hat{j} \in \{1,2,3\} \\ -1 & \text{if } \hat{i} = \hat{j} = 0 \end{matrix} \right\} = \eta_{\hat{i}\hat{j}}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

duality:

$$\hat{e}^{\hat{i}} \cdot \hat{e}_{\hat{j}} = \delta^{\hat{i}}{}_{\hat{j}}$$

$$\vec{V} = v^{\hat{i}} \hat{e}_{\hat{i}}$$

$$\hat{e}^{\hat{i}} \cdot \vec{V} = v^{\hat{j}} \qquad \hat{e}_{\hat{j}} = \eta_{\hat{j}\hat{i}} \hat{e}^{\hat{i}} \qquad \eta^{\hat{i}\hat{j}} \text{ inverse of } \eta_{\hat{i}\hat{j}}$$

$A^{\hat{i}\hat{j}}$ raise and lower with $\eta$ too

Can't always get both orthonormal properties (ortho and normal) and duality

$$\vec{V} = v^{\hat{i}} \hat{e}_{\hat{i}}$$

component have physical meaning, not basis vector if orthonormal

$$= v^{i} \vec{b}_{i}$$

takes same magnitude

only normalizable if $g_{ij}$ is diagonal

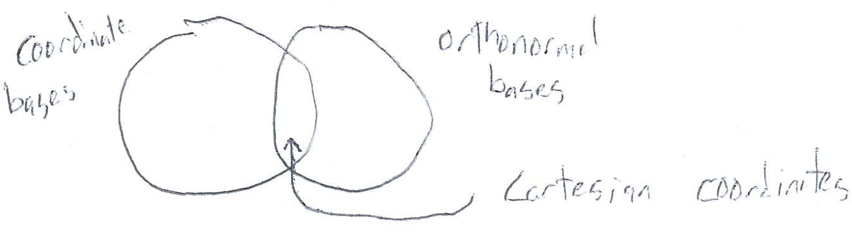
$$g_{ij} = \vec{b}_i \cdot \vec{b}_j$$

<u>Mantra</u> interpret in an orthonormal basis
calculate in a coordinate basis

in general $g$ is non-diagonal

$g$ is diagonal $\leftrightarrow$ orthogonal curvilinear coordinates

$g = \eta \leftrightarrow$ orthonormal coordinates



$\nabla \phi$

$$\nabla f = \frac{\partial f}{\partial \vec{R}} = \frac{\partial q^j}{\partial \vec{R}} \frac{\partial f}{\partial q^j} = \vec{b}^j \partial_j f$$

e.g. Cylindrical coordinates in 3D

$$r = \sqrt{x^2 + y^2}\ , \quad \phi = \arctan\left(\frac{y}{x}\right)\ , \quad z = z$$

$$r \geq 0\ , \quad 0 \leq \phi < 2\pi\ , \quad -\infty < z < \infty$$

$$\vec{b}^r = \nabla r = \nabla \sqrt{x^2+y^2} = \cos\phi\, \hat{x} + \sin\phi\, \hat{y} = \hat{r}$$

$$\vec{b}^z = \nabla z = \hat{z}$$

$$\vec{b}^\phi = \nabla \phi = \frac{1}{r}\left(-\sin\phi\, \hat{x} + \cos\phi\, \hat{y}\right) = \frac{1}{r}\hat{\phi}$$

$$\vec{b}_r \cdot \vec{b}_z = 0 \qquad \vec{b}_r \cdot \vec{b}_\phi = 0 \qquad \vec{b}_\phi \cdot \vec{b}_z = 0$$

orthogonal but not orthonormal

$$\vec{b}_r = \frac{\partial \vec{R}}{\partial r} = \hat{r}$$

$$\vec{b}_\phi = \frac{\partial \vec{R}}{\partial \phi} = r\hat{\phi}$$

$$\vec{b}_z = \frac{\partial \vec{R}}{\partial z} = \hat{z}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{\partial f}{\partial z}\hat{z} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\phi}$$

in Cartesian Coordinates, $g_{\mu\nu} = \eta_{\mu\nu}$

Lorentz Transformation

Postulates of Special Relativity

(0) inertial frames exist

(1) laws of nature are the same in all inertial frames

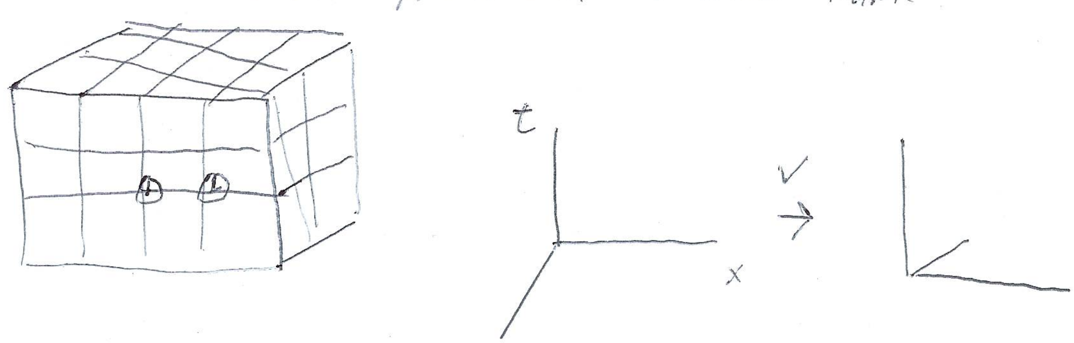
$\Longrightarrow$ Space and time are homogeneous and isotropic

(2) Speed of light is finite and independent of the motion of the source (or the state of motion of the observer)

or

(2') in every inertial frame there is a finite universal limiting speed c for physical interactions or information transfer

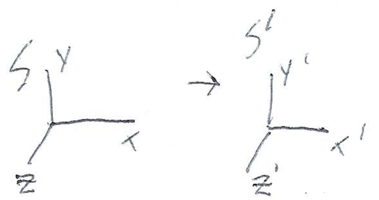
Difference coordinates system and reference frame




Homogeneity and continuity $\longrightarrow$ transformation must be affine

Newton's 1st Law

standard configuration




S' moves along x axis with velocity v relative to S

$x = x' = 0$
$y = y' = 0$ at $t = t' = 0$
$z = z' = 0$

Parallel Transformation

$$x' = Ax + By + Cz + Dt + E$$

$$A = A(v) \text{ , etc.}$$

if $x = vt$, $y = 0$, $z = 0$ then $x' = 0$, $y' = 0$, $z' = 0$ $\Rightarrow D = -Av$

$$x = A(x - vt) + By + Cz$$

Flipping $y \Rightarrow -y$

$$A(x-vt) + B(-y) + Cz = A(x-vt) + By + Cz$$

$$B = -B$$
$$B = 0$$

Flipping $z \rightarrow -z$ $\Rightarrow$ $C = -C \Rightarrow C = 0$

$$x = A(x - vt)$$

Flip $x \rightarrow -x$

$$-x' = A(-v)(-x - (-v)t)$$
$$x' = A(-v)(x - vt)$$
$$= A(v)(x - vt)$$

$$A(-v) = A(v)$$

reverse perspective as to which is "moving"

$$x = A(x' + vt')$$
$$x' = A(x - vt)$$

both simultaneously true

Galileo: Assuming $t = t'$

$$x' = A(x - vt) \qquad x' = \frac{1}{A}x - vt$$

$$(A - \tfrac{1}{A})x - (A - 1)vt = 0$$

$A = 1$ Galilean transformation $t' = t$

$$x' = x - vt$$

Einstein if $x = ct$

$x' = ct'$

$$ct = ct' = A(c-v) + A(c+v)t'$$

$$\Rightarrow A^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$A = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

reduces to identity when $v \to 0$