Phys 110B 02Sep20

$\vec{V} = v^i b_i = \sum_i v^i b_i$

choose basis so coefficients transform as a rank-1 tensor

coordinate basis

coordinate basis

$\vec{R}(q^1, q^2, q^3, \ldots)$

coordinate curves

vary 1 coordinate and leave others fixed

coordinate basis vectors = tangent vectors to coordinate curves

$$\vec{b}_i = \frac{\partial \vec{R}}{\partial q^i}$$ ith coordinate basis vector

$$d\vec{R} = \frac{\partial \vec{R}}{\partial q^i} dq^i = \vec{b}_i \, dq^i$$

$$ds^2 = d\vec{R} \cdot d\vec{R} = (\vec{b}_i \, dq^i) \cdot (\vec{b}_j \, dq^j) = (\vec{b}_i \cdot \vec{b}_j) \, dq^i dq^j$$

spacetime interval

metric tensor

covariant Rank-2 tensor

symmetric

$$g_{ij} = \vec{b}_i \cdot \vec{b}_j$$

metric tensor

metric spaces g is positive definite

pseudo-Riemanian manifold g is no longer positive definite

$$\vec{V} = v^i \vec{b}_i$$

$$\vec{V} \cdot \vec{b}_j = v^i \vec{b}_i \cdot \vec{b}_j = v^i g_{ij}$$ if coordinates are orthonormal g_{ij} is diagonal

dual coordinate basis

define $$\underbrace{\vec{b}^j}_{\text{dual basis}} \cdot \underbrace{\vec{b}_i}_{\text{coordinate basis}} = \delta^j{}_i = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

$\vec V = v^i b_i$

$\vec b^j \cdot \vec V = \vec b^j \cdot (v^i \vec b_i) = v^i (\vec b^j \cdot \vec b_i) = v^i \delta^j_i = v^j$

$\vec V = v_j \vec b^j$

$\vec b_i \cdot \vec V = \vec b_i \cdot (v_j \vec b^j) = v_j \vec b^j \cdot \vec b_i = v_j \delta^j_i = v_i$

$v^i \vec b_i = \vec V = v_j \vec b^j$ some relationship needed, is metric tensor

define g^{ik} so that $g^{ik} g_{kj} = \delta^i_j$

$\vec b^i = g^{ij} \vec b_j$ $\quad$ $\vec b_i = g_{ij} \vec b^j$

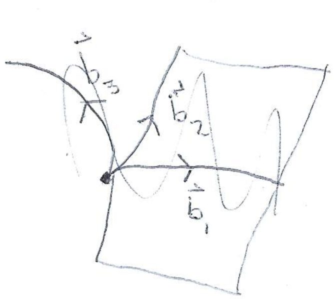
$\vec V = v_i \vec b^i = (v_i g^{ij}) \vec b_j = v^j \vec b_j$

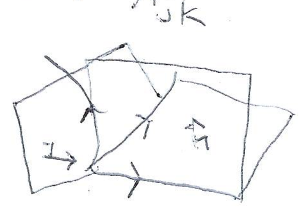
$v^j = g^{ji} v_i$ $\quad$ $v_j = g_{ji} v^i$

$g_{ij} A^j{}_k = A_{ik}$ $\quad$ $\vec x^T A \vec y$

$\cancel{A}\ \overleftrightarrow{A}(\vec b_j, \vec b_k) = A_{jk}$

$\overleftrightarrow{A}(v^j \vec b_j, v^k \vec b_k) = v^j v^k A_{jk}$





coordinate surface
= level surfaces of each coordinate

$\vec b^j = \nabla q^j = \frac{\partial q^j}{\partial \vec R}$

$\vec b^i \cdot \vec b_j = \frac{\partial q^i}{\partial \vec R} \cdot \frac{\partial \vec R}{\partial q^j} = \frac{\partial q^i}{\partial q^j} = \delta^i_j$

$g_{ij} = \vec{b}_i \cdot \vec{b}_j$

g^{ij} = inverse

$\cancel{g^{ij}} = \quad g^i{}_j = g^{ik} g_{kj} = \delta^i{}_j$

$g^{ij} = g^{jl} g^i{}_l = g^{il} \delta^i{}_l = g^{ji} = g^{ij}$

$\vec{u} \cdot \vec{w} = (u^i \vec{b}_i) \cdot (w^j \vec{b}_j) = u^i w^j (\vec{b}_i \cdot \vec{b}_j) = g_{ij} u^i w^j$

$\vec{u} \cdot \vec{w} = u^i g_{ij} w^j = w^j g_{ji} u^i = u^j w_j = u_i w^i$

rank-0 tensor

scalar

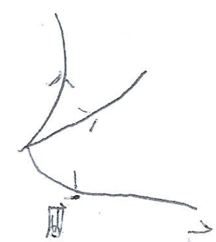
$q \to q'$

$\vec{b}'_i = \frac{\partial \vec{R}}{\partial q'^i} = \frac{\partial q^j}{\partial q'^i} \frac{\partial \vec{R}}{\partial q^j} = \frac{\partial q^j}{\partial q'^i} b_j$

$\vec{u} \cdot \vec{w} = u_i w^i = u'_i w'^i$

transform with inverse and regular Jacobian

$u'_i =$



$\vec{b}_i(q)$

q^j

Christoffel symbols not tensors

not tensors $\vec{b}^k \cdot \frac{\partial}{\partial q^j} \vec{b}_i = \Gamma^k_{ij}$

Define Orthonormal bases

$\vec{b}^k(\vec{b}_j) = \vec{b}^k \cdot \vec{b}_j$

$(w_k \vec{b}^k)(v^j \vec{b}_j) = w_k v^j b^k \cdot b_j = w_i v^i$