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1.7

High-energy muons traversing matter lose energy according to

$$-\frac{1}{\rho}\frac{dE}{dx} \approx a + bE$$

where a is due to ionisation energy loss and b is due to the bremsstrahlung and e^+e^- pair-production processes. For standard rock, taken to have $A = 22$, $Z = 11$, and $\rho = 2.65gcm^{-3}$, the parameters a and b depend only weakly on the muon energy and have values $a \approx 2.5MeVg^{-1}cm^2$ and $b \approx 3.5x10^{-6}g^{-1}cm^2$.

1.7.a

At what muon energy are the ionisation and bremsstrahlung/pair-production processes equally important?

For the two processes to be equally important, the a term and the b term should be equal ($a = bE$). Since we know a and b, all we need to do is move those terms to one side to find our muon energy, E. $E = a/b = 71000$ MeV = 710 GeV.

1.7.b

Approximately how far does a 100 GeV cosmic-ray muon propagate in rock?

We can rearrange the provided equation using some tricks of the trade that would make any mathematician shudder to get it into an easier to integrate form.

$$\frac{dE}{a+bE} = -\rho dx$$

$$\frac{1}{b}\ln{(a+bE)}\Big|_E^0 = -\rho x\Big|_0^d$$

$$\frac{-1}{b\rho}(\ln{a} - \ln{(a+bE)}) = d$$

$$\frac{\ln{(1+\frac{bE}{a})}}{b\rho} = d$$

$$d = 14000cm = 140m$$

What we end up with is about 140 meters of penetration which is a pretty impressive distance.