Analytic Mechanics

Todd Hirtler

21Jan20

Overview

Classical Mechanics describes the motion of material objects. If we were to describe the motion of Earth from some place close-by, it would be very difficult to describe. If we to consider its motion from some distance far away, such as the Sun, we can approximate the Earth as a point particle. This makes its motion a lot easier to describe with minimal lost information due to the approximation.

Motion can be described using 3 spacial dimensions and 1 temporal dimension. In describing motion using these coordinates, we can consider the temporal dimension variable being our /textbfindependent variable and the spacial dimension variables being out /textbfdependent variables.

Newtonian Mechanics will deal with vectors describe the nature of motion of the system we are trying to understand. **Lagrangian** and **Hamiltonian Mechanics** will base their descriptions of the system using scalar quantities. Scalar quantities are a lot more easy to work with than vectors, so these different ways of describing the system will be very useful.

Kinematics describes motion by determining position, velocity and acceleration as a function of time without finding out the cause of motion.

Coordinate Systems

Coordinate Systems are what we use to describe positions of system and its possible many components. Here are listed 3 of the most popular.

1. Cartesian

$$\boldsymbol{r} = x\hat{x} + y\hat{y} + z\hat{z}$$
$$= x_1\hat{x_1} + x_2\hat{x_2} + x_3\hat{x_3}$$

$$= \sum_{i=1}^{3} x_i \hat{x}_i = x_i \hat{x}_i$$

2. Cylindrical

$$x = r\cos(\phi)$$

$$y = r\sin(\phi)$$

$$z = z$$

$\hat{r}$ and $\hat{\phi}$ are position dependent since their direction is not constant over all space.

3. Spherical

$$x = r\sin(\theta)\cos(\phi)$$

$$y = r\sin(\theta)\sin(\phi)$$

$$z = r\cos(\theta)$$

$\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ are all function of θ, ϕ and t.